Variable Smoothing in Bayesian Spatial Modelling

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Abstract: We discuss an extension of the Markov random field (MRF) for spatial smoothing with small area data first introduced in Besag *et al.* (1991). We allow a smoothing parameter to be associated with each small area, thus allowing the level of smoothing to vary in space. We present the essentials of such a method, describing the implementation in WinBUGS, and illustrate the variable smoothing on both simulated and real data sets.

Keywords: Bayesian hierarchical modelling; small-area data; spatial smoothing.

1 Introduction

We introduce an adapted form of the Markov random field (MRF) for spatial smoothing with small area data first introduced in Besag *et al.* (1991)—the so-called *BYM scheme*. We relax the restriction of having a single smoothing parameter controlling smoothness over the entire map, a constraint which may or may not be appropriate in any given application. The BYM MRF is defined in terms of *differences* of random effects, and hence we define the variance on each pairwise difference to be the sum of "contributions" from each area—regarding these contributions as variable smoothing parameters of the scheme. We can define conditional moments of the spatial random effects for this new scheme, as with the BYM original. It can be shown, by appeal to a proof in Ghosh *et al.* (1998), that the scheme does give rise to proper posterior distributions.

2 Model Fitting

We take what is, in effect, an Empirical Bayes approach to fitting models via a two-stage procedure. In the first stage, we fit a model using the standard BYM scheme. This provides estimates of spatial random effects, and we use these to gain information on the local levels of variability in different regions of the map. In the second stage, we fit a model using the adapted scheme allowing for variable smoothing.

3 Simulation Study

We assess the performance of the adapted scheme via simulated data from Poisson models. We consider simulation models with (a) a gradient in the smoothing parameters and (b) patches of differing mean levels having no spatially-structured effects. For (a), the spatial random effects are recovered more accurately with our variable smoothing scheme, in terms of mean square errors; DIC statistics are also favourable. For (b), we observe improved local bias characteristics using the variable smoothing scheme, as illustrated by Figures 1 and 2 which show the per-area estimated squared bias in the random effects.
Even without seeing the map containing the original data with the patches of higher mean levels, it is easy to identify the boundaries of those patches from Figure 1. However, these areas are much more difficult to discern with the variable smoothing squared bias map of Figure 2. The simulated data in this study were generated from simple Poisson regression models, including random offsets and covariate values. There was nothing in the fitting model to account for the differing mean levels in (b), so the random effects were attempting to explain this feature of the data; note that the average squared bias in the random effects of the variable smoothing analysis was around half that of the BYM analysis.
In summary, it seems that using an overly complex spatial scheme seems safer than using too simple a scheme; this view is the result of studying bias and DIC properties on simulated data sets from models of varying complexity.

4 Application: Grazing impacts of herbivores

For a practical illustration, we present data from a study into the impacts of grazing animals on vegetation in upland Scotland (Brewer *et al.*, 2004). The existence of feeding stations for sheep results in very locally-defined patches of high impact areas, for example, and the BYM scheme appears to oversmooth in such instances. Application of the adapted scheme to these data results in fitted maps which display less smoothing of areas such as the feeding stations, but slightly more smoothing in regions of open moorland with similar vegetation. It seems clear that the adapted scheme has allowed the smoothness to change to reflect different features of the landscape.

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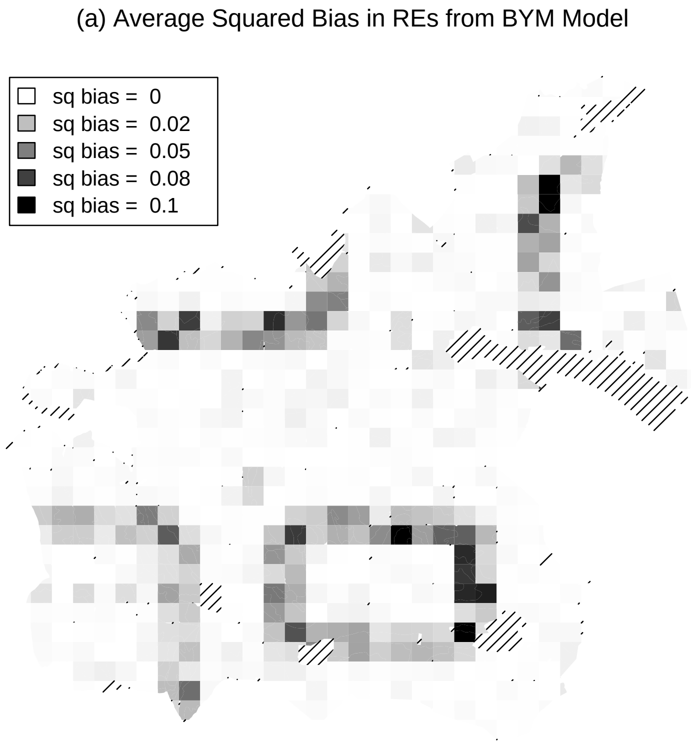

FIGURE 1. Map of the estimated squared biases in the random effects for the spatial smoothing analyses using the BYM scheme.

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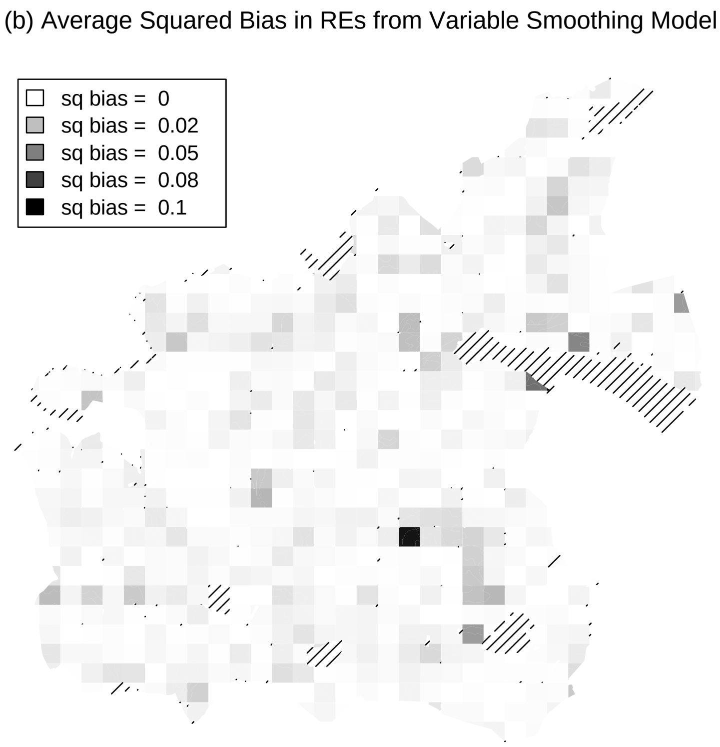

FIGURE 2. Map of the estimated squared biases in the random effects for the spatial smoothing analyses using the variable smoothing model.