A deeper look at some properties of space-time covariance functions

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Abstract: We present space-time covariance models which satisfy two main features: (a) they may be negative or oscillate between negative and positive values, and (b) they would preferably allow for an easy interpretation. In particular, we show some properties of correlation models which can attain negative values, being at the same time compactly supported.

Keywords: Negative covariances; Space-time covariance functions; Space-time Descente; Space-time Montée

1 Introduction

The modelling of biological or environmental phenomena which evolve in space and time has been one of the most important challenges over the last years. As pointed out by Janauer (2001) in an excellent paper, there is a strong interaction between biology and hydrology in the establishment, fluctuation and limitation of the aquatic environment in space and time. For instance, the analysis of the spatio-temporal distribution of flow is strongly related to the study of current velocities and turbulences. Particularly, some studies of the spatio-temporal correlation of turbulences have already been done for the evaluation of kinematical and dynamical effects (Yakhot et al., 1989). In another notable work in Levinson et al. (1984), the problem of space-time statistics of turbulences is faced up. In the merely temporal framework, it is worth citing Xu et al. (2003a,b). On the other hand, in the spatial context, Shkarofsky (1968) emphasised the fact that in the study of turbulences it is often desirable to have covariance models allowing for negative values or oscillations from positive to negative values as the Euclidean distance tends to infinite. Unfortunately, most of previously proposed spatio-temporal covariance models in literature (see, among many others, Cressie and Huang, 1999; Gneiting, 2002a; Christakos, 2000) are positive in the whole domain, so they are not useful for this purpose. This strong motivation prompted our research. We are looking for space-time covariance models satisfying two main features:

1 They may be negative or oscillate between negative and positive values.
They would preferably allow for an easy interpretation. We believe it is reasonable to select a class of models satisfying property 2, and then answer, if possible, to the natural question: can we obtain negative covariances starting from a class which is easy-to-implement and interpretable? This problem was faced, in the merely spatial case, by Ma (2005) who proposed linear combinations of spatial covariances and variograms, where at least one of the weights can be set to be negative. With a similar approach, Gregori et al. (2006) face the problem in space-time, proposing the so-called *Sum of the Products* model, for which they obtain linear combinations with negative parameters for the so obtained space-time covariances and variograms.

An additional problem which geostatisticians should take into account is computation. As pointed out in Gneiting (2002b) and in a more recent paper by Furrer et al. (2006), the use of compactly supported correlation functions induces considerable computational gains. Thus, it would be nice to have correlation models which can attain negative values, being at the same time compactly supported. This problem has been studied by Sasvári et al. (2006) in a recent paper, where the authors show that it is possible to obtain compactly supported correlation functions which can have negative values or oscillate between positive and negative ones. In particular, the authors obtain some negative correlations starting from the Golubov function (Gneiting, 2000).

The knowledge of the properties of space-time covariances and correlations can be somehow obtained as extension of well-known properties studied in the mainly spatial domain. This is the case, for instance, of the *Montée* and *Descente*, for which we refer the reader to Gneiting (1998). Porcu et al. (2006) show that, working under the assumption of isotropy between components, the natural extension to the spatio-temporal case of the *Montée* and *Descente* can be obtained. These results allow to obtain new closed forms of space-time covariances starting from other ones. In particular, they show that, starting from a covariance function defined on $\mathbb{R}^1 \times \mathbb{R}^1$, it is possible to obtain, through partial differentiation, a new class of covariances defined on $\mathbb{R}^3 \times \mathbb{R}$, having a sort of hole effect.

In this work we shall expose these last advances, that can be very important for a better knowledge and analysis of space-time random fields. Also, we shall propose a simulation study of spatial Gaussian real valued random fields defined on $\mathbb{R}^2$ and whose associated correlation function can have negative values in some parts of its domain. Estimations of the parameters are provided through composite likelihood approaches, where we show that this method can have considerable computational gains and thus can be very useful for the analysis of large spatial datasets.

We shall end with a discussion about possible future interesting research directions for the analysis of space-time data.
References


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