Child Care Costs, Female Labor Force Participation and Public Policy

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Abstract

Over the last decades, female labour force participation has increased in all high income OECD countries. The participation rates of mothers, however, are significantly lower than those of other women. As a result, there is a concern of how work and family life can be balanced. At the same time, measures of child well being are worse for families where one of the parents does not work and for one parent families. One of the barriers that women face at the time of working is to find child care for their children. Furthermore, there exists a wide variation in the success of countries in promoting a balanced work and family life across the OECD. Which type of policies can help families to reconcile work and family? In order to answer this question I build an overlapping generations model of endogenous household formation and fertility choice where households invest in their children and divide their time between work, rearing children and leisure. I introduce child care as a barrier for mothers’ participation in the labor market by making mothers purchase one unit of child care for each hour that they work. I first study a benchmark economy that features the current labor market (child care subsidies) and welfare policies (family benefits) in the U.S. Then I carry out two experiments to assess the effects of a change in U.S. family policy towards a Swedish style family policy. The first experiment consists of increasing the available child care subsidy. In the second experiment I give higher family benefits to all households with children. Interestingly, both policies increase investment in children. While child care subsidies achieve this by increasing married female labor force participation, with more generous welfare payments married female labor force participation declines. Child care subsidies, however, are more effective in alleviating poverty among households with children.

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1 Introduction

Over the last decades, female labor force participation has increased in all high income OECD countries. In 1970, 44% of women in the OECD were participating in the labor market, while in 2006, 70% of women between 25 and 54 years of age were participating in the labor market. In Germany, 62 per cent of women aged between 15 and 56 are currently employed, in the UK and in the US this number is around 67 per cent, while 72% of women are employed in Sweden. As more women go into employment, there is a concern about how work and family life can be balanced. Participation rates of mothers are significantly lower than those of other women. Participation rates of mothers with small children (younger than 2 years) range from 36 per cent in Germany to 72 per cent in Sweden, with 53 per cent of mothers working in the UK and 54 per cent in the US. The fact that mothers participate less in the labor market suggests that there are some barriers that make it difficult to work while having children. High child care costs is one of the barriers preventing mothers to participate in the labor market. Child care costs in the US can amount up to 30% of the average income of a low-income family, while in Sweden, child care costs represent 3% of average family income. Different participation rates across countries also suggest that these barriers seem to vary across countries. While in Sweden participation rates of mothers with children aged below 2 remain as high as for women overall, in Germany participation of mothers decreases to 36.1 per cent from 62 per cent for women as a whole and in the US participation rates of mothers decreases by 12 percentage points compared to all women.

Most OECD governments have on their agenda policies directed at balancing work and family life. In the US, the American Recovery and Reinvestment Act includes several policies aimed at providing easier access to child care for working families. Under this act, the Child and Dependant Care tax and funds for other programs that provide child care such as Head Start will be increased. The UK government has the goal of eradicating child poverty by 2020.

Governments are not only concerned about facilitating women’s participation in the labor market, but they are also concerned about how they affect children’s well being. Poverty in households with children is one of the measures used to assess the well-being of children. The US poverty in households with children is one of the highest among the OECD countries (17%), while the Swedish poverty rate is one of the lowest (3.6%).

Thus, some governments are more successful than others in promoting high female employment and achieving better measures of children’s well-being at the same time. Different policies directed at balancing work and family might account for this. Table I summarizes differences in
family policy between the US and Sweden. Sweden is one of the OECD countries with the lowest poverty rate among households with children, while at the same time it has one of the highest female labor force participation even among mothers. Conversely, the US has one of the highest poverty rates in the OECD and female participation in the labor market decreases significantly for mothers. At the same time, the US and Sweden differ in their family policy across several dimensions. While child care access is universal and almost free of charge in Sweden, the US only provides small subsidies to child care. This leads to differences in spending on preschool education. Sweden spends around 2 per cent of its GDP on the education of children below the age of 3, while the US spends 0.4 per cent of its GDP.

Table I: Policies in US and Sweden

<table>
<thead>
<tr>
<th></th>
<th>Sweden</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family benefits</td>
<td>Universal child benefit</td>
<td>Means-tested benefit</td>
</tr>
<tr>
<td>Child care</td>
<td>Universal free access</td>
<td>Small subsidies</td>
</tr>
<tr>
<td>Family benefits</td>
<td>3.21% of GDP</td>
<td>1.27% of GDP</td>
</tr>
<tr>
<td>Pre-school funding</td>
<td>1.9% of GDP</td>
<td>0.4% of GDP</td>
</tr>
<tr>
<td>Income Tax rate</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>

Source: OECD, 2007

The main contribution of my paper is to assess the extent to which these differences in policies account for differences in female labor force participation, fertility rates and children’s well-being. To this end, I present an overlapping generations model (OLG) of endogenous household formation and fertility choice where households invest in their children and divide their time between work, child-rearing and leisure. Child care is costly and for each hour that mothers work, they have to purchase child care. I first study a benchmark economy that features the current labor market (child care subsidies) and welfare policies (family benefits) in the U.S. The model economy is able to capture the key aspects of the data regarding marital status of the population, level and timing of fertility, well-being of children and welfare dependency. Then I carry out two experiments to assess the effects of a change in U.S. family policy towards a Swedish style family policy. The first experiment consists of increasing the available child care subsidy. I consider an increase such that the U.S. child care subsidy is comparable to the OECD average subsidy. With
an increase in child care subsidy, which lowers the cost of children, the participation of women in the labor market increases significantly, especially for single women and women with lower productivity. Fertility also increases. However, the fraction of poor households with children declines and this translates into higher human capital levels for children, especially for children of poorer and single women. The second experiment consists of giving higher family benefits to all households with children. Contrary to the results yielded by the first experiment, labor market participation of mothers decreases significantly. Fertility as well as the human capital investment received by children increases under higher benefits as they did under higher child care subsidies. However, education for children living in single mother households is higher under the child care subsidy. Moreover, in contrast to child care subsidies, poverty among households with children increases if family benefits are used as a policy instrument.

**Background**  Child care costs could be one of the barriers preventing women with children from participating in the labor market.\(^1\) When mothers work, they face the problem of who is going to take care of their children. This is especially true for young children. Formal child care can take a sizeable share of family income. In table II, I present monthly child care fees per two year olds attending full time early child care education as a percentage of average worker wage income for several OECD countries. In Sweden child care costs represent one of the lowest fraction of income in the OECD, while in the US and the UK, child care costs take 20 per cent or more of the average wage income of a worker. In table III, I present monthly child care costs as a percentage of family income for different household’s characteristics for the US. Families with a monthly income in the $3000- $4499 bracket face child care costs that amount to 10 per cent of their income. If the family lives under the poverty line, child care costs amount to 30 per cent of their income. These costs are substantial and they could be preventing mothers from joining the labor force.

\(^1\)See Del Boca (2002), Del Boca and Sauer (2009), Baker, Milligan and Gruber (2008), and Haan and Wrohlich (2009), Nicodemo and Waldman (2009).
Table II: OECD Monthly Child care cost, 2004

<table>
<thead>
<tr>
<th>Country</th>
<th>Child care cost as % of average earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>4.5</td>
</tr>
<tr>
<td>Germany</td>
<td>9.1</td>
</tr>
<tr>
<td>UK</td>
<td>24.7</td>
</tr>
<tr>
<td>US</td>
<td>19.5</td>
</tr>
</tbody>
</table>

Source: OECD, 2007

Table III: Child care costs, 2005

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>USA as % of family income</th>
<th>USA as % of mother’s income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family income, monthly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt;$1,500</td>
<td>32.7</td>
<td>34.0</td>
</tr>
<tr>
<td>$1,500 - $2,999</td>
<td>16.2</td>
<td>21.4</td>
</tr>
<tr>
<td>$3,000 - $4,499</td>
<td>10.8</td>
<td>18.8</td>
</tr>
<tr>
<td>$4,500 and over</td>
<td>5.0</td>
<td>1.1</td>
</tr>
<tr>
<td>Work status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full time employed</td>
<td>6.8</td>
<td>13.3</td>
</tr>
<tr>
<td>Poverty status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below poverty line</td>
<td>29.2</td>
<td>41.6</td>
</tr>
<tr>
<td>Marital status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>5.6</td>
<td>12.9</td>
</tr>
<tr>
<td>Divorced</td>
<td>10.7</td>
<td>12.2</td>
</tr>
<tr>
<td>Never married</td>
<td>12.3</td>
<td>17.9</td>
</tr>
</tbody>
</table>

Source: US Census Bureau, 2005

Moreover, employment of mothers and household structure seem to have a significant impact on children’s well being. A measure of children’s well-being often considered is the poverty rate in households with children. Table IV presents child poverty rates for different types of households in Sweden, the US and Germany. Poverty rates of households with children vary widely across
countries. In Sweden, the proportion of households with children with an income below 50 per cent of the median income is 3.6 per cent while in the US, it is 17.6 per cent. In Germany 13.16 per cent of households with children are poor. One-parent families as well as couple families where only one parent works suffer higher poverty rates.

Table IV: Poverty rate in households with children

<table>
<thead>
<tr>
<th></th>
<th>Households all ages</th>
<th>Singles age&lt;6</th>
<th>Couples Not working</th>
<th>Couples working 1 worker</th>
<th>Couples 2 workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>3.6</td>
<td>-</td>
<td>18,15</td>
<td>6,29</td>
<td>13,65</td>
</tr>
<tr>
<td>USA</td>
<td>17.6</td>
<td>21</td>
<td>91,53</td>
<td>36,20</td>
<td>27,01</td>
</tr>
<tr>
<td>Germany</td>
<td>13.16</td>
<td>-</td>
<td>56.1</td>
<td>26.34</td>
<td>5.7</td>
</tr>
<tr>
<td>UK</td>
<td>8.9</td>
<td>-</td>
<td>39</td>
<td>7</td>
<td>27</td>
</tr>
</tbody>
</table>

Source: OECD, 2007

Child development and well being are major concerns in the OECD countries. As noted in Kamerman et al (2003), children living in poor households are more prone to experience bad health and bad school outcomes. These might in turn lead to worse outcomes in adult life. Cunha, Heckman, Lochner and Masterov (2005) show that important differences in the skills of children across family types appear at early ages and are persistent (see also Almond and Currie 2010 for a recent review). They look at Peabody Individual Achievement Test in Math (PIAT Math), which measures age-appropriate math knowledge and they find that there are large gaps. Once they control for maternal education, cognitive ability and family structure (broken home or not), the gap across racial and income groups is reduced. Higher levels of family resources in a child formative years are associated with higher quality education and, therefore, the fact that a family is credit constraint and it does not have adequate resources is an important issue for the children’s human capital accumulation.

Household structure is an important factor to take into account when considering children’s well-being. Not only one parent households seem to exhibit higher poverty rates but there is substantial evidence indicating that family background is a key determinant of success in adulthood, see Haveman and Wolfe (1995) for a review. McLanahan and Sandefur (1994) also point out that children raised by a single parent perform worse in adulthood than those in a two parent household. Those adults who have spent their childhood in a single-parent household
have a greater risk of being poor, experiencing teenage pregnancy and participating in criminal activities. Neal and Johnson (1996) argue that differences in skills are accountable for most of the black-white wage gap and in turn the skill gap can be traced back to observable differences in family background. They show that the black-white wage gap is mostly accounted for by differences in human capital accumulation (as measured by the Armed Forces Qualification Test) before the ages of 16-18.

Finally, it is important to note that household structure in most industrialized countries has changed over the last decades. With divorce rates and out-of-wedlock births on the rise, the two parent model of family unit is no longer the norm, and a significant fraction of children live in one-parent households in most OECD countries (26 per cent in US, 21 percent in Sweden, 13.4 per cent in Germany). Therefore household structure should be taken into account when analyzing the balance between family life and work.

**Related Literature** This paper is related to recent papers that use quantitative equilibrium models of family formation to evaluate public policy. In a general equilibrium framework, Greenwood, Guner and Knowles (2000) investigate the effect of the rise in the generosity of welfare payments on the rising incidence of single motherhood.² Guner and Knowles (2009) compare welfare policies in Canada to the ones in the U.S. and check whether differences in welfare policy can account for the higher rates of single parenthood observed in the U.S. The authors use a model of endogenous marriage, and divorce decisions and endogenous fertility choice.

Domeij and Klein (2008) also formally model the need for child care for employed mothers. In a framework of stochastic dynamic life-cycle model of household decision making with exogenous fertility, the authors try to answer the question of whether subsidies on day care raise welfare by encouraging women to work. Bernal and Fruttero (2006) focus on the effect of parental leave policy on female labor supply decisions, intrahousehold allocation and investment in children’s human capital. They find that introducing maternity leave will increase female labor force participation, time spent with child and human capital investments in children. Erosa, Fuster and Restuccia (2010) develop a model of fertility choice and labor market decisions to study the role of parental leave policies. Bick (2010) investigates the relationship between the provision of child care and female labor force participation and fertility in a life cycle model. He finds that the lack of child care is in fact a barrier to female labor force participation and it reduces

²See also see, among others, Erosa, Fuster and Restuccia (2010), Attanassio, Low and Sanchez-Marcos (2008), Brown and Flinn (2006), Casarico and Sommacal (2008), and Del Boca, Flinn and Wiswall (2010).
fertility. In my paper, I also allow for policies to have an effect on the level of human capital that children acquire and I account for the effect of a change in policy on the poverty faced by households with children.

2 Environment

The economy is populated by overlapping generations of individuals who live for two periods as children and for three periods as adults. I refer to the first, second and third period of adulthood as young, middle aged and old, respectively. There is a continuum of men and women of unit mass in each age group. Agents differ in productivity. Let the productivity of women be denoted by \( x \in X = \{x_1, \ldots, x_N\} \), and that of men by \( z \in Z = \{z_1, \ldots, z_N\} \). The productivity in the second and third period depends on the productivity of the previous period in the following way

\[
\Pr [x' = x_j \mid x = x_i] = \pi_x (x_j \mid x_i) \quad \text{and} \quad \Pr [z' = z_j \mid z = z_i] = \pi_z (z_j \mid z_i).
\]

When individuals become young adults, they observe their productivity and they randomly meet their potential spouses from the same cohort. Potential couples draw a match quality \( \gamma \in G = \{\gamma_1, \ldots, \gamma_Q\} \) with probability \( \Gamma (\gamma) \). Both partners observe their types and match quality and they marry if both agree to do so. Otherwise they remain single for that period.

At the beginning of individuals’ middle age, their new productivity is realized. Agents who are married also observe their spouses’ productivity and their match quality. They face a probability \( p_\gamma \) of having the same match quality as last period and with probability \( (1 - p_\gamma) \) they have to draw a new match from the same distribution as above. Then, they decide whether to stay married or get divorced. Divorced individuals have to wait one period to be able to remarry. Agents who have remained single in the previous period meet and, as before, depending on their productivities and match quality, decide whether to marry or not. In the last period of their lives, agents who are married decide whether to stay married or divorce and those who were single or divorced in the last period, meet potential spouses and face the choice to marry.

After marriage market decisions have been taken, couples and single females have to decide how many children to have, how women’s time is split between working, taking care of children and leisure and how much income to spend on children’s education. Men are assumed to work a fixed amount of time \( n \), whether they are married or single. This assumption is made to concentrate on the decisions of women. Women can have children when they are young and middle aged. Let \( K \) denote the total number of children in the household. Agents receive utility
from consumption, leisure, from educating their children, \( e \) and the number of children. For women, utility per period is given by

\[
F(c, e, K, 1 - l - t, \gamma) = \begin{cases} 
\nu^c(c) + \nu^e(Q(e, K)) + \nu^t(1 - l - t - \psi_f K) - \gamma, & \text{if married,} \\
\nu^c(c) + \nu^e(Q(e, K)) + \nu^t(1 - l - t - \psi_f K), & \text{if single}
\end{cases}
\]

If a woman is married, she enjoys the value of the match quality, \( \gamma \). Women have to decide how much to work \( l \) and in case they have children the time they dedicate to them, \( t \). They also have to incur a fixed time cost for each child, \( \psi_f \). \( Q(.,.) \) is a function representing the quality-quantity trade-off between having more children and giving each child more education.

Utility for a man is given by

\[
M(c, e, K, \gamma) = \begin{cases} 
u^c(c) + u^e(Q(e, K)) + u^n(1 - n - \psi_m K) - \gamma, & \text{if married,} \\
u^c(c) + u^n(1 - n), & \text{if single.}
\end{cases}
\]

It is similar to that of a woman if he is married. A father also incurs a fixed time cost, but he does not spend any time taking care of children. If he is single or divorced, he does not receive any utility from his children. A couple takes decisions by maximizing a weighted average of the individuals’ utilities, where \( \mu \) is the weight given to the wife’s utility. Children spend two periods with their parents after which they become young adults. Children take no decisions. During childhood, children need to go to private child care for as long as the mother works out of home. For each unit of time that the mother is at work, the child must be a unit of time in private child care at a cost \( \delta l \). Upon divorce, children are assumed to remain with their mothers and their fathers do not receive any utility from them nor do they have to pay any child support nor alimony.

Education per child depends on the time that mothers spend with the child \( t \), the cost of sending the child to private child care \( \delta l \) and the monetary cost of raising a child, \( g \). The education received by a child is denoted by

\[
e = \Xi(t, g, l, K)
\]

The probability of different productivity realizations when young is affected by the education received as a child. It is denoted by

\[
\Pr[x = x_j \mid e] = \Pi^x(x_j \mid e) \quad \text{and} \quad \Pr[z = z_j \mid e] = \Pi^z(z_j \mid e),
\]

where \( e = e_{-1} + e_{-2} \) is the total education received by an individual during her/his childhood.
There is a government that collects taxes and it uses them to pay out subsidies and transfers and for government consumption $G$. Agents have to pay income taxes $\tau(Y)$ and they receive transfers $T(K,Y)$ that might depend on the number of children, $K$ and income, $Y$. The government also offers a subsidy to private child care, $\omega$.

3 Value functions

I start with the problem faced by agents when they are old and go backwards to the problem of a young agent.

3.1 Single/ Divorced Old

I start with the problem of a single/ divorced woman. A woman who is single when old could have been married when young and divorced when middle aged or she could have never been married. A divorced woman might have married in the first or second period and divorced in the last period of her life. Old women cannot have children but any children they had when middle aged still live with them. The value function of single life for an old woman is given by

$$S^f_3(x_i,K) = \max_{c,e,g,l,t} F(c,e,l,K,1-l-t,0),$$

subject to

$$c = \Psi(1,K)(x_i l - \tau(x_i l) - \delta l (1 - \omega) + T(K,Y) - g),$$

and

$$e = \Xi(t,g,l,K).$$

The problem facing a divorced woman is the same as that of a single woman. $\Psi(a,K)$ is the adult-equivalent size of a household with $a$ adults and $K$ children. Income of the household is equal to labor income plus transfers minus income taxes. A single woman has to pay for private child care $\delta$ for each hour she is working and in addition she faces other kinds of expenditure that affect the education of her children $g$.

The problem of a man is simple as he just works a fixed time $n$

$$S^m_3(z_j) = \max_c M(c,0),$$

3 Government consumption is not productive and it does not give any utility.
subject to
\[ c = z_j n - \tau (z_j n) . \]
A divorced old man has the same problem as that of a single old man.

### 3.2 Married Old

An old married couple could have been newly formed, i.e. they matched and married when old, or they could have been married for one or two periods already. The value of being married in the last period is given by

\[
V_3(x_i, z_j, K, \gamma) = \max_{c, e, g, l, t} \mu F (c, e, K, 1 - l - t, \gamma) + (1 - \mu) M (c, e, K, \gamma) ,
\]
subject to
\[
c = \Psi (2, K) [x_i l + z_j n - \tau (x_i l + z_j n) - \delta l (1 - \omega) + T (K, Y) - g] ,
\]
and
\[
e = \Xi (t, g, l, K) .
\]

An old married couple upon observing their productivities and their match quality, has to decide if they want to remain married or divorce. They will stay married if and only if

\[
W_3(x_i, z_j, K, \gamma) \geq S^f_3 (x_i, K) \text{ and } H_3(x_i, z_j, K, \gamma) \geq S^m_3 (z_j) \text{ where }
\]

\[
W_3(x_i, z_j, K, \gamma) = F^* (c, e, K, 1 - l - t, \gamma)
\]
and \( F^* (c, e, K, 1 - l - t, \gamma) \) denotes the utility of a woman evaluated at the decision taken by the couple and

\[
H_3(x_i, z_j, \gamma, K) = M^* (c, e, K, \gamma)
\]
where \( M^* (c, e, K, \gamma) \) denotes the utility of a man evaluated at the decision taken by the couple.

The decision of staying married is represented by the following indicator function

\[
I^d_{3,j} = \begin{cases} 
1, & \text{if } W_3(x_i, z_j, K, \gamma) \geq S^f_3 (x, k) , \\
0, & \text{otherwise,}
\end{cases}
\]
and

\[
I^d_{3,m} = \begin{cases} 
1, & \text{if } H_3(x_i, z_j, K, \gamma) \geq S^m_3 (z_j) , \\
0, & \text{otherwise,}
\end{cases}
\]
for a woman and a man respectively.
3.3 Single/ Divorced Middle aged

A single middle aged woman was also single when young while the divorced female was married. A single woman starts her middle aged with labor productivity \( x_i \) and \( k \) children from when she was young. She has to decide on how much to consume, how many children to have this period, how to split her time between leisure, work and time with her children and how much money she will spend on her children. Let \( b \in \{0, 1, ..., N_k\} \) be the number of number of children the woman can give birth to this period. She has \( k \) children who were born while she was young. Therefore, the total number of children she will have this period will be the number of children born while she was young and the number of children born this period. Let the total of children in the household be denoted by \( K \). A woman values her current utility and the expected continuation value for the next period. In order to form expectations about their future status women need to know the distribution of single men in the third period, \( \Omega_3(z) \). In the next period, the productivity of a middle aged woman evolves according to the process \( \pi_x(., x_i) \) and she has to decide whether to remain single or marry. She will be matched to a single man of type \( z_t \) from the distribution \( \Omega_3(z) \) and they will draw a match quality \( q \) with probability \( \Gamma(\gamma_q) \). The value of not being married this period is given by

\[
S^f_2(x_i, k) = \max_{c, e, l, t, b} F(c, e, K, 1 - l - t, 0) + \beta \sum_{p=1}^N \pi_x(x_p \mid x_i) \times \\
\max\{S^f_3(x_p, k'), \sum_{l=1}^N \Omega_3(z_l) \sum_{q=1}^Q \Gamma(\gamma_q) W_3(x_p, z_l, k', \gamma_q) I^q_{3,m}\},
\]

subject to

\[
c = \Psi(1, K)(x_i l - \tau(x_i l) + T(K, Y) - \delta l (1 - \omega) - g),
\]

\[
e = \Xi(t, g, l, K),
\]

where

\[
K = k + b \text{ and } k' = b.
\]

Men also need to know the distribution of single women next period, \( \Phi_3(x, k) \) over types and the number of children living with them. Again, men just work a fix amount of time and their
problem is the following

\[ S_m^2(z_j) = \max_c M(c) + \beta \sum_{l=1}^{N} \pi_z(z_l | z_j) \times \]

\[ \max\{ S_m^3(z_l), \sum_{j=0}^{N_k} \sum_{p=1}^{N} \Phi_3(x_p, k_j) \sum_{q=1}^{Q} \Gamma(\gamma_q) H_3(x_p, z_l, k_j, \gamma_q) \frac{I_{s,f}}{I_{3,f}} \} \],

subject to

\[ c = z_j n - \tau(z_j n) . \]

3.4 Married middle aged

There are two types of marriages in the second period. There are newly formed marriages and those that were formed in the first period of the cohort’s adult life. Couples observe their productivities and their match quality. A couple formed by a woman of type \( x_i \) and a man of type \( z_j \) with \( k \) children and a match quality \( \gamma \) has to decide on how many children to have this period \( b \), how much to spend educating the \( K \) children present in the household, \( g \), and how to split the woman’s time between work, taking care of the children and her leisure. In order to make decisions, the couple maximizes a weighted sum of expected values of marriage. The expected weighted value of marriage for the woman depends on the current utility \( F(.,.) \) and the expected value of the marriage. In the next period, the couple faces a probability \( p_{q} \) of keeping the same match quality, \( \gamma \) as in the current period. Else, they will have a match quality \( \gamma_q \) with probability \( \Gamma(\gamma_q) \) in the next period. The productivity of the woman and the man evolve stochastically according to the processes \( \pi_x(x | x_i) \) and \( \pi_z(., z_j) \) respectively and they will have \( k' \) children that are those born in the current period, \( b \), as children borne in the first period have become young adults. The continuation value of being married in the next period depends on whether the expected utility they get as a couple is greater than the utility as divorced individuals.

The expected utility of a couple formed by a woman of type \( x_i \) and a man of type \( z_j \) with \( k \)
children and a match quality $\gamma$ is

$$
V_2 (x_i, z_j, k, \gamma) =
$$

$$
\max_{c, e, t, l, b} \mu (F (c, e, K, 1 - l - t, \gamma) + \beta p_\gamma \sum_{p=1}^{N} \pi_x (x_p | x_i) \times
$$

$$
[\max \{ S_3^f (x_p, k') \sum_{l=1}^{N} \pi_z (z_l | z_j) W_3 (x_p, z_l, k', \gamma) I_{3,m} \}] + \beta (1 - p_\gamma) \sum_{k=1}^{N} \pi_x (x_k | x_i) \times
$$

$$
[\max \{ S_3^f (x_p, k') \sum_{l=1}^{N} \pi_z (z_l | z_j) \sum_{q=1}^{Q} \Gamma (\gamma_q) W_3 (x_p, z_l, k', \gamma_q) I_{3,m} \}]
$$

$$
+ (1 - \mu) (M (c, e, K, \gamma) + \beta p_\gamma \sum_{l=1}^{N} \pi_z (z_l | z_j) \times
$$

$$
[\max \{ S_3^m (z_l) \sum_{p=1}^{N} \pi_x (x_p | x_i) H_3 (x_p, z_l, k', \gamma) I_{3,f} \}] + \beta (1 - p_\gamma) \sum_{l=1}^{N} \pi_z (z_l | z_j) \times
$$

$$
[\max \{ S_3^m (z_l) \sum_{p=1}^{N} \pi_x (x_p | x_i) \sum_{q=1}^{Q} \Gamma (\gamma_q) H_3 (x_p, z_l, k', \gamma_q) I_{3,f} \}]
$$

subject to

$$
c = \Psi (2, K) [r (x_i l + z_j n) + T (K, Y) - \delta l (1 - \omega) - g],
$$

and

$$
e = \Xi (t, g, l, K),
$$

where

$$
K = k + b \text{ and } k' = b.
$$

They decide to remain married if and only if they both agree to do so. For the woman, the value of remaining in the marriage should be greater or equal than divorcing, $W_2 (x_i, z_j, k, \gamma) \geq S_2^f (x_i, k)$ and for the man this condition should also hold, $H_2 (x_i, z_j, k, \gamma) \geq S_2^m (z_l)$ where

$$
W_2 (x_i, z_j, k, \gamma) = F^* (c, e, K, 1 - l - t, \gamma) + \beta p_\gamma \sum_{p=1}^{N} \pi_x (x_p | x_i) \times
$$

$$
[\max \{ S_3^f (x_p, k') \sum_{l=1}^{N} \pi_z (z_l | z_j) W_3 (x_p, z_l, k', \gamma) I_{3,m} \}] + \beta (1 - p_\gamma) \sum_{k=1}^{N} \pi_x (x_k | x_i) \times
$$

$$
[\max \{ S_3^f (x_p, k') \sum_{l=1}^{N} \pi_z (z_l | z_j) \sum_{q=1}^{Q} \Gamma (\gamma_q) W_3 (x_p, z_l, k', \gamma_q) I_{3,m} \}].
$$
\( W_2(x_i, z_j, k, \gamma) \) is the value of marriage for a woman evaluated at the optimal decisions made by the couple and \( H_2(x_i, z_j, k, \gamma) \) is the value of marriage for a man evaluated at the decisions made by the couple,

\[
H_2(x_i, z_j, k, \gamma) = M^* (c, e, K, \gamma) + \beta p_y \sum_{l=1}^{N} \pi_z (z_l | z_j) \times \\ [\max\{S^m_3 (z_l), \sum_{p=1}^{N} \pi_x (x_p | x_i) H_3 (x_p, z_l, k', \gamma) I^d_{3,f} \}] + \beta (1 - p_y) \sum_{l=1}^{N} \pi_z (z_l | z_j) \times \\ [\max\{S^m_3 (z_l), \sum_{p=1}^{N} \pi_x (x_p | x_i) \sum_{q=1}^{Q} \Gamma (\gamma_q) H_3 (x_p, z_l, k', \gamma_q) I^d_{3,f} \}].
\]

The decision is represented by the following indicator function

\[
I^d_{2,f} = \begin{cases} 
1, & \text{if } W_2(x_i, z_j, k, \gamma) \geq S^f_2 (x_i, k), \\
0, & \text{otherwise},
\end{cases}
\]

and

\[
I^d_{2,m} = \begin{cases} 
1, & \text{if } H_2(x_i, z_j, k, \gamma) \geq S^m_2 (z_j), \\
0, & \text{otherwise},
\end{cases}
\]

for a woman and a man respectively. Therefore, if the value of being married is greater or equal than the value of being single for a woman, then she wants to be married and the indicator function \( I^d_{2,f} \) takes the value one. For the marriage to take place, the man also needs to get a higher value from marriage than from being a bachelor, if this is so, the indicator function \( I^d_{2,m} \) takes also the value 1.

### 3.5 Single Young

When young, a single woman has to decide how much to work, how many children to have, the time she will spend taking care of them, how much private child care to purchase and how much to spend on her children’s education in order to maximize her expected lifetime utility. She also needs to know how many single men there will be in the second period, as there are \( N \) different productivity types of men she expects to be matched to a single man with productivity \( z_l \) with
probability $\Omega_2 (z_l)$. A young single woman’s value function is

$$S_1^f (x_i) = \max_{c,e,g,l,t,b} \left[ F (c, e, b, 1 - l - t, 0) + \beta \sum_{p=1}^{N} \pi_x (x_p \mid x_i) \times \right.$$

$$\max \{ S_2^f (x_p, k), \sum_{l=1}^{N} \Omega_2 (z_l) \sum_{q=1}^{Q} W_2 (x_p, z_l, k, \gamma_q) \Gamma (\gamma_q) I_{2,m} \},$$

subject to

$$c = \Psi (1, K) (x_i l - \tau (x_i l) + T (K, Y) - \delta l (1 - \omega) - g),$$

and

$$e = \Xi (t, g, l, K)$$

where

$$k = b.$$

For the young single man, the problem is the following

$$S_1^m (z_j) = \max_{c} M (c, 0) + \beta \sum_{l=1}^{N} \pi_z (z_l \mid z_j) \times$$

$$\max \{ S_2^m (z_l), \sum_{j=0}^{N_k} \sum_{p=1}^{N} \Phi_2 (x_p, k_j) \sum_{q=1}^{Q} \Gamma (\gamma_q) H_2 (x_p, z_l, k_j, \gamma_q) I_{2,f} \},$$

subject to

$$e = z_j n - \tau (zn).$$
3.6 Married Young

When a young couple meets, they have to decide whether to marry or to remain single. The expected value of being married when young, \( V_1 (x_i, z_j, \gamma) \) is given by

\[
V_1 (x_i, z_j, \gamma) = \max_{c,e,g,l,t,b} \mu(F(c, e, b, 1 - l - t, \gamma) + \beta p_\gamma \sum_{p=1}^{N} \pi_x (x_p | x_i) \times \\
\times \max \{ \frac{S_2^f (x_p, k)}{x}, \sum_{l=1}^{N} \pi_z (z_l | z_j) W_2 (x_p, z_l, k, \gamma) I_{2,m}^d \} + \\
+ \beta (1 - p_\gamma) \sum_{p=1}^{N} \pi_x (x_p | x_i) \times \\
\times \max \{ \frac{S_2^f (x_p, k)}{x}, \sum_{l=1}^{N} \pi_z (z_l | z_j) \sum_{q=1}^{Q} \Gamma (\gamma_q) W_2 (x_p, z_l, k, \gamma_q) I_{2,m}^d \} + \\
+ (1 - \mu) (M(c, e, k, \gamma) + \beta p_\gamma \sum_{l=1}^{N} \pi_z (z_l | z_j) \times \\
\times \max \{ \frac{S_2^m (z_l)}{x}, \sum_{p=1}^{N} \pi_x (x_p | x_i) H_2 (x_p, z_l, k, \gamma) I_{2,f}^d \} + \\
+ \beta (1 - p_\gamma) \sum_{l=1}^{N} \pi_z (z_l | z_j) \times \\
\times \max \{ \frac{S_2^m (z_l)}{x}, \sum_{p=1}^{N} \pi_x (x_p | x_i) \sum_{q=1}^{Q} \Gamma (\gamma_q) H_2 (x_p, z_l, k, \gamma_q) I_{2,f}^d \} ,
\]

subject to

\[
c = \Psi(2, K) (x_i l + z_j n - \tau (x_i l + z_j n) + T (K, J) - \delta l (1 - \omega) - g) ,
\]

and

\[
e = \Xi(t, g, l, K) ,
\]

where

\[
k = b.
\]
A couple will get married if and only if \( W_1(x_i, z_j, \gamma) \geq S_1^f(x) \) and \( H_1(x_i, z_j, \gamma) \geq S_1^m(z_j) \), where

\[
W_1(x_i, z_j, \gamma) = F(c, e, b, 1 - l - t, \gamma) + \beta p_{\gamma} \sum_{p=1}^{N} \pi_x(x_p \mid x_i) \times \left( \max\{S_2^f(x_p, k), \sum_{l=1}^{N} \pi_z(z_l \mid z_j) W_2(x_p, z_l, k, \gamma) I_{2,m}^d \} \right) + \\
+ \beta (1 - p_{\gamma}) \sum_{p=1}^{N} \pi_x(x_p \mid x_i) \times \left( \max\{S_2^f(x_p, k), \sum_{l=1}^{N} \pi_z(z_l \mid z_j) \sum_{q=1}^{Q} \Gamma(\gamma_q) W_2(x_p, z_l, k, \gamma_q) I_{2,m}^d \} \right)
\]

and

\[
H_1(x_i, z_j, \gamma) = M(c, e, b, \gamma) + \beta p_{\gamma} \sum_{l=1}^{N} \pi_z(z_l \mid z_j) \times \left( \max\{S_2^m(z_l), \sum_{p=1}^{N} \pi_x(x_p \mid x_i) H_2(x_p, z_l, k, \gamma) I_{2,f}^d \} \right) \\
+ \beta (1 - p_{\gamma}) \sum_{l=1}^{N} \pi_z(z_l \mid z_j) \times \left( \max\{S_2^m(z_l), \sum_{p=1}^{N} \pi_x(x_p \mid x_i) \sum_{q=1}^{Q} \Gamma(\gamma_q) H_2(x_p, z_l, k, \gamma_q) I_{2,f}^d \} \right).
\]

The decision of marrying is represented by the indicator function

\[
I_{1,f}^d = \begin{cases} 
1, & \text{if } W_1(x_i, z_j, \gamma) \geq S_1^f(x_i), \\
0, & \text{otherwise},
\end{cases}
\]

and

\[
I_{1,m}^d = \begin{cases} 
1, & \text{if } H_1(x_i, z_j, \gamma) \geq S_1^m(z_j), \\
0, & \text{otherwise}.
\end{cases}
\]

4 Stationary Equilibrium

In order to characterize the equilibrium the optimal decision rules as well as the stationary distributions of women and men have to be determined and the government budget has to be balanced every period.
Let’s denote optimal decision rules for the old single woman as follows \( L_{Sf} (x, k) \) is the labor decision, \( T_{Sf} (x, k) \) is the time spent with her children, \( G_{Sf} (x, k) \) is the amount of money spent on the education of her children, \( E_{Sf} (x, k) \) is the education given to her children and \( C_{Sf} (x, k) \) is the level of consumption. \( C_{Sm} (z) \) is the optimal consumption of an old single male and the optimal decisions for the old married couple are: \( L_{V} (x, z, k, \gamma) \) is the labor decision for the wife, \( T_{V} (x, z, k, \gamma) \) is the time the wife spends with their children, \( G_{V} (x, z, k, \gamma) \) is the amount of money the couple spends on the education of their children, \( E_{V} (x, z, k, \gamma) \) is the education they give to their children and \( C_{V} (x, z, k, \gamma) \) is the household consumption level. For the second period, the decision rules are \( L_{Sf} (x, k) ; T_{Sf} (x, k) ; G_{Sf} (x, k) ; E_{Sf} (x, k) ; C_{Sf} (x, k) \), \( G_{Sm} (z) \) for single men and \( L_{V} (x, z, k, \gamma) ; T_{V} (x, z, k, \gamma) ; G_{V} (x, z, k, \gamma) ; E_{V} (x, z, k, \gamma) ; C_{V} (x, z, k, \gamma) \), and \( C_{V} (x, z, k, \gamma) \).

**DEFINITION.** Given the government policies, \( \{\tau, T, \omega, G\} \), a stationary equilibrium is a set of decision rules defined above, the number of children that single and married women have \( K_{Sf}, K_{V} \) respectively, and distributions \( \Phi_{1} (x) , \Phi_{2} (x, k) , \Phi_{3} (x, k) \) and \( \Omega_{1} (z) , \Omega_{2} (z) , \Omega_{3} (z) \) such that:

- Given the government policy and the distributions, the rules above are the solutions to the value functions described above.
- The distributions \( \Phi_{1} (x) , \Phi_{2} (x, k) , \Phi_{3} (x, k) \) and \( \Omega_{1} (z) , \Omega_{2} (z) , \Omega_{3} (z) \) are stationary distributions which are consistent with the decision rules.
- Government budget is balanced

\[
\tau Y = G + P
\]

where \( Y \) is the economy’s income and \( P \) is the amount of family benefits and child care subsidies handed out to families.\(^4\)

\(^4\)See Appendix for an explicit form of \( P \).
5 Quantitative Analysis

I start by presenting the explicit functional forms of the utility functions, the education function, the process mapping human capital received during childhood and the process determining the productivity from one period to the next. I keep functional forms quite general in order to allow for flexibility in the calibration section. Women and men have the same preferences for consumption, children and leisure. Their utility is separable in consumption, children and leisure. The curvature parameters are $\sigma_c$ for the consumption, $\sigma_k$ for the utility referring to children and $\sigma_l$ is the curvature parameter for the utility of leisure. There are two weighting parameters. The weight parameter for children is $\phi_k$ and the weight for leisure in the utility function is $\phi_l$. The weight for consumption is normalized to 1. The match quality does not interact with decisions on how to split time, how much to consume and how many children to have but it affects the marriage decisions. The utility of a woman $F(c, e, K, 1 - l - t, \gamma)$ is

$$F(c, e, K, 1 - l - t, \gamma) = \begin{cases} \frac{1}{1 - \sigma_c} c^{1 - \sigma_c} + \phi_k \frac{1}{1 - \sigma_k} Q^{1 - \sigma_k} + \phi_l \frac{1}{1 - \sigma_l} (1 - l - t - \psi_j K)^{1 - \sigma_l} - \gamma, & \text{if married,} \\ \frac{1}{1 - \sigma_c} c^{1 - \sigma_c} + \phi_k \frac{1}{1 - \sigma_k} Q^{1 - \sigma_k} + \phi_l \frac{1}{1 - \sigma_l} (1 - l - t - \psi_j K)^{1 - \sigma_l} - \gamma, & \text{otherwise.} \end{cases}$$

For a man the utility function $M(c, e, K, \gamma)$ is

$$M(c, e, K, \gamma) = \begin{cases} \frac{1}{1 - \sigma_c} c^{1 - \sigma_c} + \phi_k \frac{1}{1 - \sigma_k} Q^{1 - \sigma_k} + \phi_l \frac{1}{1 - \sigma_l} (1 - n - \psi_m K)^{1 - \sigma_l} - \gamma & \text{if married,} \\ \frac{1}{1 - \sigma_c} c^{1 - \sigma_c} + \phi_l \frac{1}{1 - \sigma_l} (1 - n)^{1 - \sigma_l}, & \text{otherwise.} \end{cases}$$

Households get utility from the number of children in the household and from the level of education that these children have received. Following Becker and Tomes (1976), I assume that there is a trade off between the number of children households can have and the level of human capital that households can provide for their children. For the quantity quality trade off of children, a Cobb Douglas specification has been chosen as other papers in the literature.\footnote{Greenwood et al. (2000). Other specifications do not change results considerably.} The share of education in the production function of child quality is equal to $\lambda$.

$$Q = e^\lambda K^{1 - \lambda}$$
The education production function depends on the time spent taking care of the children, $t$, and the amount of money spent on the education of the children. Money and time are assume to have unit elasticity of substitution. The share of time spent taking care of children in the education production function is equal to $\theta$. Money can be spent in two different ways. If the mother works $l$, children have to spend that amount of time in private child care and pay an hourly cost of $\delta$. The cost of private child care is productive in the sense that it educates children. The other expenditure parents face is $g$ which represents any other type of expenditure related to the education of children. The chosen functional form to represent the relationship between expenditure on private child care and other education related expenditure is CES. The elasticity of substitution between these two types of expenditures is equal to $\frac{1}{1-\rho}$ and the weight of expenditures $g$ is represented by $\alpha$. This functional form is flexible enough to capture the degree of substitutability between these two different types of expenditure, and is given by

$$e = \Xi(t, g, l, K) = \frac{1}{K} \left[ (\alpha g^\rho + (1 - \alpha) (\delta l)^\rho)^{\frac{1}{\rho}} \right]^{\theta} t^{1-\theta}.$$ 

Children receive education over two periods, so when they become young adults the human capital accumulated over their childhood maps into their initial productivity level. Both women and men initial productivities are drawn from the same distribution $\Pi(., .)$

$$\Pr[x = x_i | e] = \Pi(x_i | e)$$

and

$$\Pr[z = z_j | e] = \Pi(z_j | e).$$

I assume that $\Pi(x_i | e)$ and $\Pi(z_j | e)$ are discrete approximations to log normal distributions. Therefore $\Pi(., e)$ is a discrete approximation to a log normal distribution with mean $\varepsilon_1 e^{z_2}$ and standard deviation $\sigma_z$.

After the first period as young adults productivities for women and men evolve stochastically according to the following

$$\Pr[x' = x_j | x = x_i] = \pi_x(x_j | x_i) \quad \text{and} \quad \Pr[z' = z_j | z = z_i] = \pi_z(z_j | z_i),$$

I assume that the distributions are discrete approximations of an AR(1) process in logarithms. The persistence parameter for women $\pi_x$ and the one for men $\pi_z$ are assumed to be the same and equal to $\pi$. Thus the productivity level of a woman next period $x'$ given that her productivity was $x_i$ this period is a draw from a lognormal distribution with mean $\mu_x (1 - \pi) + \pi \ln x_i$ and standard
deviation $\sigma_x\sqrt{(1-\pi^2)}$. And for a man with productivity level $z_j$ this period, the productivity level in next period $z'$ is a draw from a lognormal distribution with mean $\mu_z (1 - \pi) + \pi \ln z_j$ and standard deviation $\sigma_z \sqrt{(1-\pi^2)}$.

5.1 Calibration

I start by calibrating the benchmark economy to match several statistics of the US economy. The calibration strategy is the following. First I fix some parameters based on available information and I introduce the parameters that I calibrate to match several US data statistics. Finally I introduce the policy instruments characterizing US family policy. Some of these parameters will be calibrated while others will come directly from the US family policy. Most of the data statistics are obtained from the American Community Survey (2007) from now on ACS.

The parameters that are set a priori are the discount factor $\beta$, the weight of the wife in the couples valuation of marriage $\mu$, the weekly hours worked by men $n$, the parameters related to the productivity distribution over individuals $\{\mu_x, \mu_z, \sigma_x, \sigma_z\}$, the parameters mapping human capital to initial productivities $\{\varepsilon_1, \varepsilon_2, \sigma_\varepsilon\}$, the fixed time cost from having children for women and men $\psi$.

One model period is equivalent to 10 years. The discount factor, $\beta$ is set to match a 4% yearly interest rate. The weight given to the wife in the household’s utility $\mu$, is equal to 0.5 to give same weight in the household to the woman and the man. I assume that all men work the same fixed amount of time to concentrate on the problem of a woman. In the US, men work on average 44 hours per week. I allow the total disposable hours of an individual in the economy to be 100. Therefore the amount of time worked by a man, $n$ is equal to 0.44. In the model economy, women (and men) differ in their productivities. In the US, labor productivity is distributed as a log normal across individuals. I discretize the distribution to obtain different productivity levels. I estimate the mean and standard deviation of the productivity distribution from the ACS and obtain the following means, $\mu_x = 2.717$ and $\mu_z = 2.983$ for women and men and standard deviations, $\sigma_x = 0.72$ and $\sigma_z = 0.723$ for women and men respectively. The parameters mapping human capital levels to initial productivities are set such that the initial distributions of young women and men productivities are consistent with labor productivity distributions observed in the US data. Thus the value of $\varepsilon_1$ is 19, the value of $\varepsilon_2$ is 0.04 and the standard deviation $\sigma_\varepsilon$ is set to 0.7. The time cost per child for men and women is assumed to be the same and equal to 8 per cent of time. This a mid-value that can be found in empirical studies, see Erosa et al,
Thus $\psi$ is set to 0.080.

The utility, education, productivity and matching parameters, the hourly cost of child care and the level of transfers given out in the economy are obtained by matching model moments to data. There are 15 parameters and 16 data moments. Parameters to be estimated are preference parameters from the utility $\{\sigma_c, \sigma_k, \phi_k, \sigma_l, \phi_l\}$ related to consumption, children and leisure, the weight on children’s education $\lambda$ in the function for parents preference for the number of children and their quality, parameters in the children’s education function $\{\alpha, \rho, \theta\}$, the persistence of productivity of men and women across periods, which they are assumed to be the same $\{\pi\}$, marriage market matching parameters $\{p_y, \gamma_l, \gamma_h, \pi_y\}$ and the hourly cost of child care $\{\delta\}$.

Even though in a general equilibrium model all parameters affect the targets, I discuss briefly the data moments that each parameter is most likely to determine. Preference parameters referring to children $\{\sigma_k, \phi_k\}$ are chosen to match the fact that 78 per cent of children are born to mothers between the ages of 25 and 35, i.e. the young period in the model and that 26 per cent of children live in households with one parent. The value for $\sigma_k$ is equal to 0 and the value for $\phi_k$ is 4.620. The preference parameter for consumption, $\{\sigma_c\}$ is calibrated to match the income ratio of single women households to married households. In the US, this ratio is 0.48 and $\sigma_c$ equals 0.412.

Average female labor force participation of single women is 80 per cent and average hours worked by single women as a fraction of those worked by married women equals 1.05. I use these data moments to match the parameters in the utility function associated with labor supply, $\{\sigma_l, \phi_l\}$. The curvature parameter $\sigma_l$ is set to 0.730 and the weight of leisure in the utility function $\phi_l$ is set to 3.580. I calibrate the persistence of productivity over time $\pi$ using average weekly hours worked by women in the US which is equal to 38 hours out of a 100 hours per week. $\pi$ is set to 0.6.

On average US women have 2.05 children. I calibrate the parameter $\lambda$ in the quantity-quality production function of children to match this fertility rate and $\lambda$ is set to 0.403. To fix the parameters of the education function $\{\alpha, \rho, \theta\}$, the following 3 data moments are used: fertility of women in the 90th percentile of the income distribution as a fraction of the fertility of women in the 10th percentile (0.95), the fraction of income spent on children by single mothers as a ratio of income spent by married couples (1.387), where the data on expenditure on children comes from the report on expenditures on children by families, USDA (2007) and the fact that children in single households receive half the education of children in two parent households (McLanahan and Sandefur, 1994). The values obtained from the calibration for the education parameters
are $\alpha = 0.810$, $\rho = 0.600$ and $\theta = 0.610$. The child care fee as a percentage of average income for a family with two working parents amounts to 19.5% of average worker income in the U.S. (OECD 2005). The hourly cost of child care ($\delta$) is set to 5.903 in order to match this fact.

Finally, the matching parameters $\{p, \gamma, \gamma_h, \pi\}$ are calibrated using marriage statistics. Remarriage of 45 to 55 year olds in the US is 14 per cent. I use this to match the persistence of the match quality $p = 0.23$. The proportion of never married in the USA is 0.245 and it is used to match the value of the low match quality, $\gamma_l$, that is set equal to 17.776. The fact that the fraction of singles between 25 and 35 years old equals 42.810 per cent of the population between 25 and 55 is used to match the high match quality, $\gamma_l$, and its value is set to -2.302. The probability of receiving a high quality match $\pi$, is matched using the fact that on average 15.500 per cent of the population is divorced in the US and the value of $\pi$ is set to 0.580.

This leaves us with the policy parameters to be determined. I assume that welfare policy takes a simple form: households with children (married or single) who earn below a threshold level $T$ receive $T$. Hence there are four policy parameters, $\tau$, $\omega$, $T$ and $\bar{T}$. I calibrate the amount of transfers $T$ to match a data moment observed in the US, while the income tax, $\tau$, and the child care subsidy, $\omega$, are set to be consistent with their US policy instruments counterparts in the model economy. $T$ denotes the transfers to families and single women with children. These transfers represent some family benefits available to low income families in the U.S. as part of the "Temporary Assistance for Needy Families" (TANF). The OECD (2005) reports family benefits received conditional on having children for the state of Michigan. They use the state of Michigan as a representative of a typical manufacturing state in the US. The benefit received is equivalent to 3 per cent of the average income of the Michigan state ($39481). I calibrate the amount of transfers a household with children receive to equal 3% of average income in my economy.

In my benchmark economy all working individuals pay a proportional tax $\tau$ on labor income. I set $\tau$ to be equal to 17% which is equivalent to the individual income tax and employment taxes collected by the government at federal and state level as a fraction of GDP (US Census Bureau, 2007, Internal Revenue service). The child care subsidy ($\omega$) is set to 5% of the child care cost to match the tax reduction associated with the use of child care of a lone parent earning average wages, (OECD 2007).

Finally, $\bar{T}$ is set to be consistent with the eligibility rules in the US. According to the Administration for Children and Family, it is the poverty threshold. The poverty threshold in the US is the same across states, however it differs by family size, age and sex of household head and different government aid programs use different measures of poverty. I use a simplified version
of the eligibility rules and I set the threshold $T$ to be the poverty line, that is 50% of the median income in the model economy.

The following tables summarize the a priori parameters, the calibrated parameter and the policy parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.675</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Weight of wife in couple’s utility</td>
<td>0.500</td>
</tr>
<tr>
<td>$n$</td>
<td>Working time of men</td>
<td>0.440</td>
</tr>
<tr>
<td>$\mu_x$</td>
<td>mean productivity of women</td>
<td>2.717</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>standard deviation of women’s productivity</td>
<td>0.720</td>
</tr>
<tr>
<td>$\mu_z$</td>
<td>mean productivity of men</td>
<td>2.983</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>standard deviation of men’s productivity</td>
<td>0.723</td>
</tr>
<tr>
<td>$\psi_f$</td>
<td>Fixed time cost per child</td>
<td>0.080</td>
</tr>
</tbody>
</table>
### Table VI: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_k$</td>
<td>weight of children</td>
<td>4.620</td>
</tr>
<tr>
<td>$\phi_l$</td>
<td>weight of leisure</td>
<td>3.580</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>preference for children</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>preference for consumption</td>
<td>0.412</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>preference for leisure</td>
<td>0.730</td>
</tr>
<tr>
<td>$\pi$</td>
<td>productivity persistence</td>
<td>0.600</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>share of education in Q function</td>
<td>0.403</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>weight of $g$ in the education function</td>
<td>0.800</td>
</tr>
<tr>
<td>$\rho$</td>
<td>elasticity parameter between $g$ and $\delta l$</td>
<td>0.600</td>
</tr>
<tr>
<td>$\theta$</td>
<td>share of expenditures in the education function</td>
<td>0.610</td>
</tr>
<tr>
<td>$p_{\gamma}$</td>
<td>probability of keeping same $\gamma$ next period</td>
<td>0.230</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>High match quality</td>
<td>-2.306</td>
</tr>
<tr>
<td>$\gamma_h$</td>
<td>Low match quality</td>
<td>17.776</td>
</tr>
<tr>
<td>$\pi_{\gamma}$</td>
<td>Probability of high match quality</td>
<td>0.580</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Hourly cost of child care</td>
<td>5.903</td>
</tr>
</tbody>
</table>

### Table VII: Policy parameters

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Family Benefits</td>
<td>0.2796</td>
</tr>
<tr>
<td>A priori</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>Income tax</td>
<td>0.170</td>
</tr>
<tr>
<td>$\bar{T}$</td>
<td>Transfers threshold</td>
<td>2.962</td>
</tr>
<tr>
<td>$\omega$</td>
<td>child care subsidy</td>
<td>0.050</td>
</tr>
</tbody>
</table>
Table VIII: Human capital mapping parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_1$</td>
<td>Mean parameter</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>Mean parameter</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>Standard deviation parameter</td>
</tr>
</tbody>
</table>

The model matches the data fairly well. Table IX, reports the moments generated by the model together with the corresponding data moment.

Table IX: Data and Model Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Moments</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fertility rate</td>
<td>2.0305</td>
<td>2.050</td>
</tr>
<tr>
<td>Single-married income ratio</td>
<td>0.535</td>
<td>0.480</td>
</tr>
<tr>
<td>Single-married hours worked ratio</td>
<td>1.254</td>
<td>1.0500</td>
</tr>
<tr>
<td>Fraction of single young</td>
<td>0.425</td>
<td>0.428</td>
</tr>
<tr>
<td>Fraction of never married</td>
<td>0.225</td>
<td>0.245</td>
</tr>
<tr>
<td>Average Divorced rate</td>
<td>0.159</td>
<td>0.155</td>
</tr>
<tr>
<td>Average hours worked</td>
<td>0.391</td>
<td>0.381</td>
</tr>
<tr>
<td>Income-Fertility relation</td>
<td>0.593</td>
<td>0.950</td>
</tr>
<tr>
<td>Average FLFP of single females</td>
<td>0.810</td>
<td>0.800</td>
</tr>
<tr>
<td>Average FLFP of married females</td>
<td>0.710</td>
<td>0.720</td>
</tr>
<tr>
<td>Single-married education ratio</td>
<td>0.34</td>
<td>0.500</td>
</tr>
<tr>
<td>Single-married expenditure ratio</td>
<td>1.345</td>
<td>1.387</td>
</tr>
<tr>
<td>Fraction of kids living with single parents</td>
<td>0.246</td>
<td>0.260</td>
</tr>
<tr>
<td>Fraction of kids born in 1st period</td>
<td>0.747</td>
<td>0.780</td>
</tr>
<tr>
<td>Remarriage rate among 45-55 year olds</td>
<td>0.12</td>
<td>0.140</td>
</tr>
<tr>
<td>Child care costs as % of average income</td>
<td>19.5</td>
<td>19.5</td>
</tr>
</tbody>
</table>
6 The Benchmark Economy

The model provides further information about the benchmark economy that has not been exploited to match the model to the data. Thus looking at the performance of the model across dimensions not calibrated to data, allows to evaluate the validity of the model to represent the US economy.

6.1 Demographic structure

In the benchmark economy, individuals live for 5 periods. The first 2 periods correspond to childhood, while the other 3 periods represent adult life. The first period of adult life is referred to as young period. For calibration purposes, the young period of the model corresponds to the age group 25 to 34 year olds in the data.

Agents live for 50 years. Childhood starts when the individual is 5 years old and ends when the individual becomes 25. Note that I assume that individuals start off their lives when they are 5. This is done in order for individuals to begin adult life at 25 as this period is when decisions on marriage, fertility and labor force participation are taken. In the US, the median age of first marriage for women is 25 and this age is also the average age at first birth. Life begins when the individual is 5 and ends at 55. From 5 to 14 years old, individuals are in their early childhood and their late childhood goes from the age of 15 to the beginning of young adulthood at 25. At 35 they begin middle aged until they become old at 45.

I have not exploited the age structure of the benchmark economy for calibration. Therefore, I use it to evaluate the goodness of the model to represent the data. First I present the marital status of the population by age group in Table X. Note that I use the fraction of single young agents, the fraction of single agents of any age and the fraction of divorced agents in the economy for calibration, the other moments were not used for calibration. The model does well in replicating the marital status by age groups. In the first period of the model, divorce is not allowed thus those who are divorced in the data are counted as still married in the model. Also, the model underestimates those individuals who remain never married when old, but on the whole
it replicates the marital patterns observed in the US economy.

Table X Marital Status of the Population

<table>
<thead>
<tr>
<th>Age group</th>
<th>Marital status</th>
<th>Never married</th>
<th>Married</th>
<th>divorced</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>25-34</td>
<td></td>
<td>42.81</td>
<td>42.45</td>
<td>48.31</td>
</tr>
<tr>
<td>35-44</td>
<td></td>
<td>19.5</td>
<td>17.86</td>
<td>64</td>
</tr>
<tr>
<td>45-54</td>
<td></td>
<td>12.65</td>
<td>0.08</td>
<td>66.73</td>
</tr>
<tr>
<td>25-54</td>
<td></td>
<td>24.5</td>
<td>22.5</td>
<td>60</td>
</tr>
</tbody>
</table>

Another statistic that is obtained from the model is the average number of children present in households with children by age group. The distribution for the US is described in table XI. The average number of children present in households headed by young single women is 1.8 in the US while in the model this number is 1. The model predicts the average number of children in married households well, however it does underestimate the number of children in single and divorced female headed households, especially when old. Even so, it replicates the qualitative structure of the average number of children in households with children in the US. Married household seem to have more children present than households headed by single females. In the US the number of children present in households is larger for the age group between 35 and 44.
years and this is also the case in the benchmark model.

Table XI. Average Number of Children in Households

<table>
<thead>
<tr>
<th>Age group</th>
<th>Average Number of children</th>
<th>Never married</th>
<th>Married</th>
<th>divorced</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-34</td>
<td>Data</td>
<td>1.8</td>
<td>2.03</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>1</td>
<td>2.35</td>
<td>-</td>
</tr>
<tr>
<td>35-44</td>
<td>Data</td>
<td>1.9</td>
<td>2.18</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>1.4</td>
<td>2.67</td>
<td>2.2</td>
</tr>
<tr>
<td>45-54</td>
<td>Data</td>
<td>1.24</td>
<td>1.46</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>0.51</td>
<td>1</td>
<td>0.46</td>
</tr>
<tr>
<td>25-54</td>
<td>Data</td>
<td>1.836</td>
<td>2.014</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>1.04</td>
<td>2.012</td>
<td>1.33</td>
</tr>
</tbody>
</table>

6.2 Who receives Welfare?

In the benchmark economy, households receive family benefits only if there are children present in the household. Therefore, single men are not entitled to family benefits. Only single/divorced women and couples with children will receive benefits. These benefits do not depend on the number of children nor the marital status of the mother. They depend on the household income. The eligibility threshold in the USA for TANF benefits differs across income, age of the children, the number of children and the marital status of the head of the family.

According to Lester and Tin (2003), in the USA, on average, 13 per cent of the whole population participates in a family means-tested benefit program in a given month. In the benchmark economy this proportion is 10 per cent, therefore, the model does quite well in
generating the proportion of individuals on welfare in the US. Table XII presents the proportion of families receiving family benefits by family type. 37 per cent of single female families received some family benefits in a given month in 1999. In the benchmark economy, this proportion is 33.3, which is reasonably close to the data. For married couples, the participation rate decreases significantly to 7 per cent, while in the model this number is lower.

Table XII

<table>
<thead>
<tr>
<th>Family type</th>
<th>Program participation rates in any means tested programs</th>
<th>1999 Data</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single female</td>
<td></td>
<td>37.1</td>
<td>33.29</td>
</tr>
<tr>
<td>Married couple</td>
<td></td>
<td>7.4</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Source: Lester and Tin, 2003

6.3 Child care costs

In the model, child care costs are a barrier for women to work. For each hour that they work, they have to pay a fee for the child to be left in care. Therefore, some women might not be able to work and pay for child care. Other women might just prefer to stay home. Therefore the financial burden of child care is different for different types of families. I will be providing some of the child care costs faced by different families in the benchmark economy. I calibrate the hourly care cost to be equal to the child care costs faced on average in the US. In the US, single female families face the highest child care costs as a percentage of family income. Also, married couple with low income spend a high proportion of their income on child care. Even when both parents are working, average costs of child care in the US can amount to almost 10 per cent of family income. In table XIII, I show the child care costs as a percentage of family income faced by different types of families in the benchmark economy.
Table XIII

<table>
<thead>
<tr>
<th>Family type</th>
<th>Child care costs as percentage of family income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>Single female</td>
<td>12</td>
</tr>
<tr>
<td>Married couple</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Source: US Census, 2005

6.4 Poor Households

Finally, I present poverty rates across different types of households with children. Poverty is defined as the proportion of household with children with an income below 50 per cent of the median income in the benchmark economy. Poverty rates in the model are higher for single female households than for married couples. If the female in the household is working, poverty rates decrease, while if the woman is not working poverty rates are higher. All single woman households are in poverty if the woman is not working and for married couples, if only the husband is working, poverty rate goes up by 13 percentage points.
<table>
<thead>
<tr>
<th>Family type</th>
<th>Poverty rates for households with children</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>single female working</td>
<td>36.20</td>
</tr>
<tr>
<td>single female not working</td>
<td>91.53</td>
</tr>
<tr>
<td>married couple, both working</td>
<td>6.17</td>
</tr>
<tr>
<td>married couple, only male working</td>
<td>27.01</td>
</tr>
</tbody>
</table>

Source: OECD, 2008

7 Experiments

I carry out two different policy experiments to investigate the effect of different family policies. Policies will be government consumption neutral. Thus the amount of tax revenue collected that is going to government consumption is the same as in the benchmark economy. I will focus on the labor market participation of women, the fertility rate and children’s well-being.

Two different kinds of experiments are performed. The first experiment consist of increasing child care subsidies. This labor market policy is designed to affect the labor market participation decision of women. The child care subsidy is conditional on working. The second experiment is a welfare type of policy. I increase the family benefits received by families with children.\(^6\)

7.1 Increase Child care subsidy

Child care costs in the US amount up to 30 per cent of average income. Therefore, this cost constitutes a considerable barrier for families to leave children in private care while at work. Subsidizing child care conditional on working could decrease that barrier. This policy is expected

\(^6\)Note that I only compare steady-states.
to increase labor supply of women and thus increase family income. I increase the child care subsidy from 5% of the hourly cost to a 65%. This might seem like a large increase, however, the OECD average child care cost is 35 per cent of the child care costs in the US. A child care subsidy of 65 per cent is the necessary subsidy to bring average child care costs in the US down to the OECD average level.

Table XV presents results of the experiment along with the results of the benchmark economy.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>$\omega = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fertility rate</td>
<td>2.035</td>
<td>2.530</td>
</tr>
<tr>
<td>Average hours worked</td>
<td>0.391</td>
<td>0.338</td>
</tr>
<tr>
<td>Average FLFP of single females</td>
<td>0.810</td>
<td>0.963</td>
</tr>
<tr>
<td>Average FLFP of married females</td>
<td>0.710</td>
<td>0.866</td>
</tr>
<tr>
<td>Poverty</td>
<td>0.353</td>
<td>0.300</td>
</tr>
<tr>
<td>Tax rate</td>
<td>0.170</td>
<td>0.250</td>
</tr>
<tr>
<td>Average education</td>
<td>0.490</td>
<td>0.518</td>
</tr>
<tr>
<td>Education received by child in one parent family</td>
<td>0.209</td>
<td>0.352</td>
</tr>
<tr>
<td>Education received by child in two parents family</td>
<td>0.632</td>
<td>0.624</td>
</tr>
</tbody>
</table>

Keeping government consumption neutral, requires an increase in income taxes from 17 per cent to 25 per cent. Increasing child care subsidies provides an incentive for women to work. The child care subsidy is in fact a subsidy to working, therefore, the model yields the expected results as both single and married women increase their labor market participation. Under the policy experiment almost all single women decide to work while 86 per cent of married women participate in the labor market.

At the same time, average hours worked by women decrease. This is due to the fact that new women going into employment have lower productivity. Thus they decide to work fewer hours than the ones who were employed before the subsidy increased. Under this policy the fertility rate increases. It is easier to have children as the cost of having to take care of them has decreased considerably.
The poverty rate in the experiment is defined as the proportion of households with children with less income than 50 per cent of the median income in the benchmark economy. Thus, the threshold for defining who is poor is kept fixed. Otherwise poverty rate is a relative measure and if median income is going up in the economy when child care subsidies are high, then poverty rate among households with children might raise due to a higher threshold. The poverty observed decreases by around 4 per cent.

Increasing child care subsidies also has an effect on the education received by children. The average education received by a child under higher child care subsidies is 25 per cent more than under the benchmark economy. This increase in the education of children affects different types of households in different proportions. Children raised in one parent households are the ones benefitting more from the increase in child care subsidies in terms of receiving more education. Under higher child care subsidies, education for a child in a one parent household is 44 per cent more than under the benchmark economy. This policy seems to promote education among children living in one parent households, who, at the same time are the ones more prone to suffer poverty and accumulate less human capital.

7.2 Increase transfers

The second policy experiment is increasing the family benefits and making them universal, i.e. there are no eligibility constraints. This policy is similar to the family policy available in Sweden, where family benefits are higher than in the US and they are available to all families with children (independent of their income level). The level of transfers (12 per cent of average income) is such that the tax needed to keep government consumption as in the benchmark economy is the same as in the previous experiment. Therefore the two experiments are comparable. Table XVI compares the benchmark economy to an economy where family benefits are higher and not means tested.
Increasing family benefits and making them available to all families with children has different effects on female labor supply than an increase in the child care subsidy. Labor market participation rates of single and married women decrease. Around 65% of women are employed. This is due to an income effect as they have now more money, some women who had to work before, decide to stay home. Fertility increases under this policy too. Women have more resources to spend on children, so they can afford to have more kids.

The poverty rate is actually slightly higher than in the benchmark economy. Even though households with children are receiving more income through family benefits, they also have less incentives to work. Therefore, women that had to work before, decide not to work any more, receiving only family benefits and thus having an income below the poverty threshold in the benchmark economy. This is especially true for single mothers whose only source of income are family benefits and earned wages.

Under this policy experiment, average education also increases. Average education is 30 percent more than under the benchmark economy. The increase in average education is similar to the case where child care subsidies are high. Looking at different types of households, one can observe that children raised in one parent households benefit less under this policy than when their mothers were receiving child care subsidies. They receive more education than in the benchmark economy but they would receive 12 percent more education if child care subsidies were...
were high instead of having generous family benefits. This policy has almost no effect on the education of children raised by two couple families.

8 Conclusion

Balancing family life (having children) and work is challenging, especially for women, who are still the primary care givers of children. Participation of women in the labor market decreases significantly for women with children because during the time that they are at work children must be left in some kind of care. Private child care is costly, amounting to almost 30 % of the average income of a low income family in the US. Therefore it might not be possible for a family to pay this cost and the mother has to stay home caring for the children. This cost might be of more concern to single parents as they do not have the income of the partner. Therefore, there is need to model labor, family structure, fertility, and the cost of private child care jointly.

In this paper I do this by building an overlapping generations model of family formation, fertility and female labor force participation. I study how the introduction of a Swedish type of family policies affects female labor participation, fertility and poverty rates. I considered two different policies: a labor market policy (increase of child care subsidy) and a welfare policy (higher family transfers).

Increasing child care subsidies leads to higher female labor force participation. The child care subsidy is like a subsidy to work. Some of the women that found the cost of child care prohibiting, they can now afford paying for child care in order to be able to work. Fertility also increases, as the family earns more, they can afford more children without compromising the education they give to each child. Finally poverty rate among households with children decreases 4 per cent.

The policy that gives higher transfers to all families with children has similar effects on fertility. However, female labor participation decreases which might not be desirable from the point of view of the government, if the government is after promoting employment among women. The poverty rate of households with children is higher when transfers are higher. Thus giving money to all families with children does not alleviate the problem of poverty in households with children. In terms of education, average education increases under both policies. However, higher child care subsidies increases the level of education received by children of single mothers more than if higher transfers are used. These children are those who tend to suffer higher poverty rates and receive less education. Thus, child care subsides have greater positive effect on the education of those more prone to receive low investment in education.
References


9 Appendix

9.1 Characterization of decision rules

9.1.1 Old problem

Let’s start with the problem of the single/divorced single old female. The three decision rules resulting from the maximization problem are given by the following system of the three first order conditions

\begin{align*}
  l & : (1/ \Psi (1, k^o) (x, l - \tau (x, l) + T (k, J) - \delta l (1 - \theta) k^o - \varphi (J) S_f - b))) \times \\
  & \times (x - \tau' (x, l) - \delta (1 - \theta) k^o) + \\
  & + \phi_1 \lambda \frac{\partial e}{\partial l} (\lambda e^\alpha + (1 - \lambda) (k^o)^\alpha) \frac{1-\alpha}{\alpha} - \phi_2 / (1 - l - t) = 0 \\
  t & : \phi_1 \lambda \frac{\partial e}{\partial t} (\lambda e^\alpha + (1 - \lambda) (k^o)^\alpha) \frac{1-\alpha}{\alpha} - \phi_2 / (1 - l - t) = 0 \\
  b & : -1/ \Psi (1, k^o) (x, l - \tau (x, l) + T (k, J) - \delta l (1 - \theta) k^o - \varphi (J) S_f - b)) + \\
  & + \phi_1 \lambda \frac{\partial e}{\partial b} (\lambda e^\alpha + (1 - \lambda) (k^o)^\alpha) \frac{1-\alpha}{\alpha} = 0
\end{align*}

The first equation is the first order condition with respect to female labor. The first term is the marginal benefit of working on income, the second term represents the effect of working on education, this effect is not clear cut as increasing labor time might decrease time spent on the child. The last term is the disutility of working. The second equation is the first order condition with respect to time spent raising the children. The first term is the marginal benefit of spending more time on the children while the second term is the marginal cost. Finally, the last equation is the first order condition with respect to the expenditure on children. The first term is the disutility of spending more on the children as income will be lower, while the second term is the marginal benefit as it will have a positive effect on the education of the children.

The first order conditions for the old married couple are similar

\begin{align*}
  l & : 1/ ((\Psi (2, k^o) (x, l - \tau (x, l) + T (k, J) - \delta l (1 - \theta) k^o - \varphi (J) S_f - b))) \times \\
  & \times (x - \tau' (x, l) - \delta (1 - \theta) k^o) + \\
  & + \phi_1 \lambda \frac{\partial e}{\partial l} (\lambda e^\alpha + (1 - \lambda) (k^o)^\alpha) \frac{1-\alpha}{\alpha} - \mu \phi_2 \varepsilon (1 - l - t)^{\varepsilon-1} = 0,
\end{align*}
\[
t : \frac{\partial e}{\partial t} (\lambda e^{\alpha} + (1 - \lambda) (k^o)^{\alpha})_{1-\alpha} - \mu \phi_2 \varepsilon (1 - l - t)^{\varepsilon - 1} = 0,
\]

\[
b : -1/ (\Psi (2, k^o) (x_l - \tau (x_l) + T (k, J) - \delta l (1 - \theta) k^o - \varphi (J) S_f - b)) + \\
+ \phi_1 \lambda \frac{\partial e}{\partial b} (\lambda e^{\alpha} + (1 - \lambda) (k^o)^{\alpha})_{1-\alpha} = 0.
\]

The single/divorced male works a fixed amount of time so he has nothing to decide, he just consumes what it is left after taxes and transfer are accounted for.

### 9.1.2 Middle aged and Young problems

Notice that the decision rules are static in the sense that they do not depend on future values of the variables. This simplifies the analysis as there is no need to know future variables to be able to take optimal decisions in a given period.

I will not present the first order conditions for the middle aged and young cohorts as they are essentially the same.