Short-run and long-run effects of banking in a New Keynesian model

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Abstract

This paper introduces endogenous capital accumulation with deposit-in-advance requirements in the banking model of Goodfriend and McCallum (2007). Impulse response functions from technology and monetary shocks show some attenuation effect due to the procyclical behavior of the marginal finance cost. In addition, a negative financial shock produces negative reactions on output, inflation and interest rates of similar size to those observed during the latest financial crisis. In the long-run analysis, we find that if banking requires 4% of total labor force there is a permanent welfare cost equivalent to 1.96% of output.

Keywords: Banking, financial attenuator, financial shock, welfare cost of banking.

JEL codes: E32, E43, E44.

1 Introduction

The current financial crisis has triggered the need for a reformulation of the New Keynesian model in a way that incorporates banking elements. Very recent papers such as Cúrdia and Woodford (2009), De Fiore and Tristani (2009), and Nolan and Thoenissen (2009) examine the implications of adding a financial sector to the New Keynesian model. Also very recently, the Journal of Monetary Economics has devoted two entire special issues to papers that study the financial crisis: the July

Most of these papers have in common the hypothesis of the "financial accelerator" that was put forth a decade ago in the seminal paper by Bernanke, Gertler and Gilchrist (1999). The financial accelerator provides a connection between the financial sector and the real sector. It assumes that the amount of loans is positively affected by the level of collateral available in the economy (bonds and the stock of capital). In good times, the value of collateral tends to rise and banks produce more loans which reduces the external finance premium and provides better opportunities for purchases of consumption and investment goods. By contrast, when the economic scenario turns gloomy the value of collateral is likely to drop and banks reduce the amount of loans because the external finance premium rises. If that reaction of the banks is severe the economy may enter a credit crunch situation like the one observed in 2008-2009. Therefore, the financial accelerator amplifies business cycle fluctuations.

This paper is another contribution to the literature on the extension of the New Keynesian models to incorporate a financial sector and banking elements. Thus, we borrow the model structure developed by Goodfriend and McCallum (2007), that hinges its banking elements in the financial accelerator of Bernanke et al. (1999), and incorporate two novel features. First, there will be variable capital accumulation in contrast with the constant capital case assumed in Goodfriend and McCallum (2007). Secondly, the deposit-in-advance requirement that households must face in their optimizing program includes total spending on consumption goods and also a partial spending on investment goods. Actually, our model introduces a \( \tau \) parameter, that takes a value between 0 and 1, to determine the fraction of investment purchases that must be backed with deposits. The setup of Goodfriend and McCallum (2007) corresponds to the particular case in which \( \tau = 0 \) because they only put consumption goods in the deposit-in-advance requirement.

The economic analysis of the consequences of a banking sector for the New Keynesian model will be carried out for a twofold perspective. On the one hand, the dynamic equations of the New Keynesian model with and without banking elements will be compared and put into play through impulse response functions. This simulation exercise will give an idea of the quantitative implications of missing banking elements for the business cycle analysis. On the other hand, the models will be solved in steady state for variety of cases that differ in the level of banking activities.
This second exercise will provide information about the long-run effects of banking elements on capital, output, consumption or welfare.

The rest of the paper is organized in seven more sections. The description of the model and the derivation of its dynamic equations are respectively done in Sections 2 and 3. A comparable model with no banking elements (frictionless finance economy) is introduced in Section 4. The calibration of the parameters is carried out in Section 5 and it is used in Section 6 to compute the impulse-response functions and discuss the results for the business cycle analysis of the short-run. Section 7 is devoted to the long-run analysis by examining the implications of alternative banking scenarios for the steady-state solution of the model. Finally, Section 8 reviews the main conclusions reached in the paper.

2 A New Keynesian model with banking

This section extends the New Keynesian model with the introduction of banking elements. Most of the banking behavior has been adapted from the model described by Goodfriend and McCallum (2007). In particular, there is loan production technology that uses collateral and banking labor as inputs, a deposit-in-advance constraint, and nominal interest rates for bonds, loans and interbank lending. One of the differences with respect to the model by Goodfriend and McCallum (2007) is the presence of endogenous capital accumulation and quadratic adjustment costs on capital changes. Households also act as bankers: the can produce loans are obtained from asset-backing collateral and banking labor. The purchases of consumption goods, \( c_t \), are carried out with deposits held in advance. Unlike Goodfriend and McCallum (2007), it is assumed here that also a fraction \( 0 < \tau < 1 \) of purchases of investment goods, \( x_t \), are carried out with nominal deposits, \( D_t \).

Moreover, there is a fraction \( V \) of total nominal deposits that must be turned over for spending

\[
P_t c_t + \tau P_t x_t = V D_t, \tag{1}
\]

where \( P_t \) is the aggregate price level. For simplicity, total deposits come from the sum of bank reserves (high-powered money, \( H_t \)) plus loans to households, \( L_t \), as indicated by the following expression

\[
D_t = H_t + L_t,
\]

\(^1\)The limit case without deposit requirement for investment goods (\( \tau = 0.0 \)) would be equivalent to the deposit-in-advance constraint assumed in Goodfriend and McCallum (2007).

\(^2\)Alternatively, there could be \( V_c \) for purchases of consumption goods and \( V_x \) for purchases of investment goods.
where bank reserves are obtained by applying the reserve coefficient to total deposits, \( H_t = rr D_t \), which implies
\[
(1 - rr)D_t = L_t. \tag{2}
\]
Combining (1) and (2) results in the following deposit-in-advance constraint expressed in real terms
\[
c_t + \tau x_t = \frac{V}{1 - rr} \frac{L_t}{P_t}.
\]
As in Goodfriend and McCallum (2007), the amount of loan production in real terms is provided by the banking technology
\[
\frac{L_t}{P_t} = L(b_{t+1}, k_{t+1}, m^d_t, A2_t, A3_t) = F(b_{t+1} + e^{A3_t \nu k_{t+1}})^\alpha (e^{A2_t m^d_t})^{1-\alpha},
\]
where \( F > 0, 0 < \nu < 1 \) and \( 0 < \alpha < 1 \) are constant parameters and \( m^d_t \) denotes the demand for labor at the bank. Goodfriend and McCallum (2007) incorporate a parameter in the loan production function that diminishes the collateral service of capital relative to bonds due to the larger monitoring effort to verify the physical condition and market value of the stock of capital. There are two banking shocks, \( A2_t \) shapes labor banking productivity and \( A3_t \) affects the productivity of the stock of capital as collateral in loan production (which it could very well indicate situations of financial stress due to overvalued or undervalued capital). The partial derivatives relating the change in the amount of loans to the change in the factors of loan production that would finance household purchases are
\[
\begin{align*}
\frac{\partial L(t)}{\partial m^d_t} &= F(b_{t+1} + e^{A3_t \nu k_{t+1}})^\alpha (1 - \alpha) (e^{A2_t m^d_t})^{-\alpha} e^{A2_t} = \frac{(1-\alpha)L(b_{t+1}, k_{t+1}, m^d_t)}{m^d_t} = \frac{(1-\alpha)(c_t+\tau x_t)}{V} \frac{1-\tau r}{V}, \\
\frac{\partial L(t)}{\partial b_{t+1}} &= F\alpha(b_{t+1} + e^{A3_t \nu k_{t+1}})^\alpha (e^{A2_t m^d_t})^{-\alpha} = \frac{\alpha L(b_{t+1}, k_{t+1}, m^d_t)}{b_{t+1} + e^{A2_t m^d_t}} = \frac{\alpha(c_t+\tau x_t)}{b_{t+1} + e^{A2_t m^d_t}} \frac{1-\tau r}{V}, \\
\frac{\partial L(t)}{\partial k_{t+1}} &= F\alpha(b_{t+1} + e^{A3_t \nu k_{t+1}})^\alpha (e^{A2_t m^d_t})^{-\alpha} e^{A3_t} = \frac{\alpha e^{A3_t \nu L(b_{t+1}, k_{t+1}, m^d_t)}}{b_{t+1} + e^{A3_t k_{t+1}}} = \frac{\alpha e^{A3_t \nu(c_t+\tau x_t)}}{b_{t+1} + e^{A3_t k_{t+1}}} \frac{1-\tau r}{V}.
\end{align*}
\]
Unlike Goodfriend and McCallum (2007), we follow Woodford (2003, page 354) to introduce an investment function that incorporates adjustment costs of changes in the stock of capital:\(^3\)
\[
x_t = I \left( \frac{k_{t+1}}{k_t} \right) k_t,
\]
where \( x_t \) is the total amount spent in period \( t \) of investment in real terms that increases the stock of capital from \( k_t \) to \( k_{t+1} \). Quoting Woodford’s book: "Increasing the capital stock to the level \( k_{t+1} \) requires investment spending in the amount \( I \left( \frac{k_{t+1}}{k_t} \right) k_t \) in period \( t \)." Hence, \( I \left( \frac{k_{t+1}}{k_t} \right) \) is a generic

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\(^3\)Sveen and Weinke (2007) also use this investment function.
convex function that determines the cost of installing next period’s capital, $k_{t+1}$, per unit of the current stock of capital $k_t$. In steady state, there is no growth and a constant stock of capital. Thus, the adjustment cost function yields in steady state $I(1) = \delta$, where $\delta$ is the rate of capital depreciation. Moreover, the first and second derivatives of the adjustment cost function in steady state are $I'(1) = 1$ and $I''(1) = \epsilon$, where $\epsilon$ is the positive measure of the curvature on the convex adjustment costs. A very high $\epsilon$ would be used to justify a constant capital in the short-run, while a number approaching to 0 for $\epsilon$ would result in a model without adjustment costs.

Households maximize intertemporal utility that depends positively on the level of consumption, $c_t$, and negatively on non-banking labor, $n_t$. They have two ways to raise labor income: working $n_t$ hours in non-banking firms and working $m^c_t - m^d_t$ net hours in the bank of other households. In both cases, they will earn the hourly market-clearing real wage $w_t$. As owners of the stock of capital, households receive the competitive real rental rate, $r^k_t$, per unit of capital lent to the firms. Households are also owners of the monopolistically competitive firms that will provide some dividends, $d_t$. Another source of income is the amount of net government transfers, $g_t$. Income is spent on purchases of consumption goods, $c_t$, on purchases of investment goods, $I\left(\frac{k_{t+1}}{k_t}\right)k_t$, on net increases of real money, $H_t/P_t - H_{t-1}/P_t$, and on net purchases of government bonds, $(1 + r^b_t)^{-1}b_{t+1} + b_t$, where $b_{t+1}$ is the amount of bonds in real terms that are bought in period $t$ to be reimbursed in $t + 1$ with a real interest rate $r^b_t$. Household preferences are defined through a constant relative risk aversion utility function, separable between consumption and labor, where future utility is brought to the current time by applying a constant discount factor per period, $\beta$.

In turn, the optimizing program of the representative household is written as follows

$$\max_{c_t,n_t,m_t,m^d_t,b_t+1,k_t+1} E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{c_{t+j}^{1-\sigma}}{1 - \sigma} - \frac{\pi_{t+j}^{1+\kappa}}{1 + \kappa} - \theta \pi_{t+j}^{1+\kappa} \right]$$

subject to current and future budget constraints

$$E_t \beta^j \left[ w_{t+j}n_{t+j} + w^m_{t+j} \left( m_{t+j} - m^d_{t+j} \right) + r^k_{t+j}k_{t+j} + d_{t+j} + g_{t+j} \right]$$

$$- c_{t+j} - I \left( \frac{k_{t+j+1}}{k_{t+j}} \right) k_{t+j} - H_t/P_t + H_{t-1}/P_t - (1 + r^b_{t+j})^{-1}b_{t+1+j} + b_{t+j} \right] = 0, \quad j = 0, 1, 2, ..., (\lambda_{t+j})$$

and to the deposit-in-advance constraints that incorporate loan production

$$E_t \beta^j \left[ c_{t+j} + \tau I \left( \frac{k_{t+j+1}}{k_{t+j}} \right) k_{t+j} - \frac{V_t}{\tau \tau} L(b_{t+1+j},k_{t+1+j},m^d_{t+j},A2_{t+j},A3_{t+j}) \right] = 0, \quad j = 0, 1, 2, ..., (\xi_{t+j})$$

There are two Lagrange multipliers, $\lambda_{t+j}$ and $\xi_{t+j}$, that respectively correspond to the budget constraint and to the deposit-in-advance constraint. The first order conditions on the six choice
variables are

\[ c_t^{-\sigma} - \lambda_t + \xi_t = 0, \quad (c_t) \]
\[-\Xi n_t - \lambda_t w_t = 0, \quad (n_t) \]
\[-\Omega m_t^e + \lambda_t w_t^m = 0, \quad (m_t) \]
\[-\lambda_t w_t - \xi_t \frac{(1-\alpha)(c_t + \tau x_t)}{m_t^d} = 0, \quad (m_t^d) \]
\[-\lambda_t (1 + r_t^{b_t})^{-1} + \beta E_t \lambda_{t+1} - \xi_t \frac{\alpha(c_t + \tau x_t)}{b_{t+1} + c_t + \tau x_t} = 0, \quad (b_t) \]
\[-\lambda_t I_t^t \left( \frac{k_t}{k_t} \right) + \beta E_t \lambda_{t+1} \left[ r_{t+1} + I_t^t \left( \frac{k_{t+2}}{k_{t+1}} \left( \frac{k_{t+2}}{k_{t+1}} \right) I(t_{k_{t+1}}) \right) \right] + \beta E_t \xi_{t+1} \left[ -r I_t^t \left( \frac{k_{t+2}}{k_{t+1}} \right) \left( \frac{k_{t+2}}{k_{t+1}} \right) + \tau I(t_{k_{t+1}}) \right] = 0, \quad (k_t) \]

where the above expressions for the partial derivatives \( \frac{\partial L}{\partial m_t^d} \), \( \frac{\partial L}{\partial b_{t+1}} \), and \( \frac{\partial L}{\partial k_{t+1}} \) were used respectively in the first order conditions for the optimal values of \( m_t^d \), \( b_{t+1} \) and \( k_{t+1} \). This model implies a relationship between its two Lagrange multipliers that can be found by rearranging terms in the first order condition of \( m_t^d \). It yields

\[ \xi_t = -\lambda_t \frac{w_t^m m_t^d}{(1-\alpha)(c_t + \tau x_t)}, \quad (\xi_t) \]

which can be substituted in the first order condition of \( c_t \) to obtain

\[ \lambda_t = \frac{c_t^{-\sigma}}{1 + \frac{w_t^m m_t^d}{(1-\alpha)(c_t + \tau x_t)}}, \quad (3) \]

The interpretation of \( \lambda_t \) is clarifying for the role of banking in the model. The shadow value of one unit of consumption, \( \lambda_t \), is its marginal utility divided by its output price. Its output price is equal to one plus the amount required to increase deposits that accommodate one extra unit of consumption. That increase in the level of deposits is required to buy one more unit of consumption. Thus, it can be noticed that the deposit-in-advance constraint raises the output price of one unit of consumption in \( w_t^m \frac{\partial m_t^d}{\partial L} \frac{\partial L}{\partial \lambda_t} \frac{\partial \lambda_t}{\partial c_t} = w_t^m \frac{m_t^d}{(1-\alpha)(c_t + \tau x_t)} \frac{V}{1-rr} \frac{1-rr}{P_t} \frac{V}{P_t} = \frac{w_t^m m_t^d}{(1-\alpha)(c_t + \tau x_t)} \).

Additional consumption requires more deposits, \( \frac{\partial D_t}{\partial c_t} = \frac{P_t}{V} \), which may be raised through some increase in loan production, \( \frac{\partial L}{\partial D_t} = \frac{1-rr}{P_t} \), which can be made by employing more banking labor, \( \frac{\partial m_t^d}{\partial L} = \frac{m_t^d}{(1-\alpha)(c_t + \tau x_t)} \frac{V}{1-rr} \), which is multiplied by the real wage to compute its output value. Hence, let us define

\[ \phi_t = \frac{w_t^m m_t^d}{(1-\alpha)(c_t + \tau x_t)} \quad (4) \]

to represent the marginal finance cost that appears in (3).
In optimizing macro models the intertemporal allocations of consumption are governed by the Euler equation that results from combining the first order conditions of $b_{t+1}$ and $c_t$. The introduction of banking features gives collateral services to bonds and capital which can be used to produce more loans as indicated by the partial derivatives $\frac{\partial L}{\partial b_{t+1}}$ and $\frac{\partial L}{\partial k_{t+1}}$ computed above. The first order condition of $b_{t+1}$ implies

$$\beta E_t \lambda_{t+1} = \lambda_t (1 + r^b_t)^{-1} + \xi_t \frac{\alpha(c_t + x_t)}{b_{t+1} + e^{A(t+1)} k_{t+1}},$$

where, using the expression of $\xi_t$ obtained above, we get

$$\beta E_t \lambda_{t+1} = \lambda_t \left(1 + r^b_t\right)^{-1} \frac{\alpha w^m m_t^d}{(1-\alpha)(b_{t+1} + e^{A(t+1)} k_{t+1})}. \quad (5)$$

The marginal financial services of bonds can be measured by the increase in real income that can be obtained with the amount of loans produced out of bonds marginal collateral services. Goodfriend and McCallum (2007) refer to this as the "liquidity service yield on bonds" and denote it as $LSY^b_t$. Such a return can be calculated as the income value of the amount of bank labor that can be saved by the use of collateral services of the bond. In formal terms, it would be $LSY^b_t = w_t \frac{\partial m^d}{\partial L} \frac{\partial L}{\partial b_{t+1}}$ that for the specific loan production function used here leads to

$$LSY^b_t = \frac{\alpha w^m m_t^d}{(1-\alpha)(b_{t+1} + e^{A(t+1)} k_{t+1})}. \quad (6)$$

The value of $LSY^b_t$ implied by (6) can be used to rewrite equation (5) as follows

$$\beta E_t \lambda_{t+1} = \lambda_t \left[\frac{1}{1 + r^b_t} - LSY^b_t\right]. \quad (7)$$

On the right-hand side of (7), the output value of the last unit of current consumption is multiplied by the term in brackets that represents the opportunity cost of buying consumption goods. There are two elements in that opportunity cost: the real interest rate on the bond $(1 + r^b_t)^{-1}$ and the marginal return of bond collateral services, $LSY^b_t$, that comes in exclusively as a consequence of having a banking sector in the model.

Inserting (3) and using (4), and also their analogous expressions for period $t + 1$, (7) is transformed into the following expression

$$\beta E_t \frac{c^-_{t+1}}{1 + \phi_{t+1}} = \frac{c^-_{t}}{1 + \phi_{t}} \left[\frac{1}{1 + r^b_t} - LSY^b_t\right], \quad (8)$$

that collapses to the standard consumption Euler equation when dropping banking elements ($\phi_t =$
\( \phi_{t+1} = LSY_t^b = 0.0 \). The loglinear approximation to (8) yields

\[
\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma} (\phi_t - E_t\phi_{t+1}) - \frac{1}{\sigma} \left( \left( r_t^b - r^b \right) + \left( LSY_t^b - LSY \right) \right),
\]

where variables topped with a hat symbol denote log deviations from the detrended levels in steady state, i.e. \( \hat{c}_t = \log \left( \frac{c_t}{c} \right) \) where \( c \) is the detrended steady-state level of consumption. Consumption dynamics are forward-looking and depend negatively on the real interest rate \( (r_t) \), the current marginal finance cost of consumption \( (\phi_t) \) and the marginal collateral service of bonds \( (LSY_t^b) \). The expected next period’s value of the marginal finance cost of consumption affects positively to current consumption.

For investment dynamics, we recall the first order condition of the stock of capital, \( (k_{t+1}) \), and substitute the relationship between Lagrange multipliers \( \xi_t = -\lambda_t\phi_t \) and \( \xi_{t+1} = -\lambda_{t+1}\phi_{t+1} \), and also the definition of the liquidity service yield on capital \( LSY_t^k = w_1 \frac{\partial m^2}{\partial L(\cdot)} \frac{\partial L(\cdot)}{\partial k_{t+1}} = \frac{\omega_\nu m^2 e^\nu}{(1-\alpha) (b_{t+1} + e^\nu \nu k_{t+1})} \) to obtain

\[
-\lambda_t (1 + \tau \phi_t) I' \left( \frac{k_{t+1}}{k_t} \right) + \beta E_t \lambda_{t+1} \left[ r_{t+1}^k + (1 + \tau \phi_{t+1}) \left( I' \left( \frac{k_{t+2}}{k_t} \right) \frac{k_{t+2}}{k_{t+1}} - I \left( \frac{k_{t+2}}{k_{t+1}} \right) \right) \right] + \lambda_t LSY_t^b = 0. 
\]

Next, equation (7) can be used in (9) to drop the Lagrange multipliers as follows

\[
- (1 + \tau \phi_t) I' \left( \frac{k_{t+1}}{k_t} \right) + \left[ \frac{1}{1 + r_t^b} - LSY_t^b \right] E_t \left[ r_{t+1}^k + (1 + \tau \phi_{t+1}) \left( I' \left( \frac{k_{t+2}}{k_t} \right) \frac{k_{t+2}}{k_{t+1}} - I \left( \frac{k_{t+2}}{k_{t+1}} \right) \right) \right] + LSY_t^k = 0. \quad (10)
\]

A log-linear approximation to (10) implies

\[
\hat{k}_{t+1} = \frac{1}{1 + \beta(1-\delta)} \hat{k}_t + \frac{\beta(1-\delta)}{1 + \beta(1-\delta)} E_t \hat{h}_{t+2} - \frac{1 + \tau \phi - LSY_t^b}{\epsilon \beta(1-\delta) (1 + \tau \phi)} \left[ \left( r_t^b - r^b \right) + \left( LSY_t^b - LSY \right) \right] + \frac{1}{\epsilon (1 + \beta(1-\delta) \tau \phi)} \left[ \beta E_t \left( r_{t+1}^k - r^k \right) + \left( LSY_t^k - LSY^k \right) \right] - \frac{1}{\epsilon (1 + \beta(1-\delta) \tau \phi)} \left[ (\phi_t - \phi) - \beta(1-\delta) E_t (\phi_{t+1} - \phi) \right].
\]

Let us turn to incorporate the economic behavior of firms as separate entities from households. There is a monopolistically competitive market where firms operate by supplying a differentiated good as initially described by Dixit and Stiglitz (1977). The optimal profit may be set by the firm

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4As a standard procedure, we used the approximation \( \log(1 + x_t) \approx x_t \) when \( x_t \) is a small number. In addition, we took the approximation \( \frac{1}{1 + r_t^b} - LSY_t^b \approx \frac{1 - LSY_t^b}{1 + r_t^b} \), observing that \( r_t^b LSY_t^b \) is negligible for being a very small number.

5Loglinearizing techniques lead to \( I \left( \frac{k_{t+1}}{k_t} \right) = \delta^{-1} (\hat{k}_{t+1} - \hat{k}_t) \) and \( I \left( \frac{k_{t+1}}{k_t} \right) = \epsilon (\hat{k}_{t+1} - \hat{k}_t) \), which were used here to obtain the loglinear capital accumulation equation.

6By contrast, Goodfriend and McCallum (2007) assume that households act as producers and there are no firms as profit-maximizing producers.
depending upon the outcome of a Calvo (1983)-type lottery. Hence, there is a constant probability, \( \chi \), that the firm cannot set the optimal price. In that case, the price would be automatically adjusted by applying an increase factor equivalent to the steady-state rate of inflation, \( \pi \). As price setters, firms face a Dixit-Stiglitz demand curve that determines the amount of output produced depending upon the relative price and aggregate output. For a representative firm, the Dixit-Stiglitz demand curve is

\[
y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} y_t,
\]

where \( y_t(i) \) is the amount of output produced by the firm at price \( P_t(i) \), given a constant elasticity of substitution \( \theta > 0 \) and the aggregate output and price level, \( y_t \) and \( P_t \). Output is produced by using a technology that employs labor and capital as inputs, and also incorporates a labor-augmenting technology shock. In formal terms, we have the production function

\[
y_t(i) = f(A_1 t n_t(i), k_t(i)),
\]

where \( A_1 \) is the labor-augmenting technology shock with a long-run growth at the trend growth rate \( \gamma \). As mentioned above, there are competitive input factors where firms can place their demand for labor and capital provided their respective market prices in terms of output, \( w_t \) and \( r_t^k \). The profit obtained by the firm \( i \) in period \( t \), expressed in units of the aggregate output, is \( P_t(i) y_t(i)/P_t - w_t n_t(i) - r_t^k k_t(i) \). Using (11), (12) and the constant discount factor \( \beta \), the optimizing program of a representative firm that can set the optimal price in period \( t \) is

\[
\max_{P_t(i), n_t(i), k_t(i)} \quad E_t \sum_{j=0}^{\infty} \beta^j \chi^j \left( \left( \frac{P_t(i)}{P_{t+j}} \right)^{1-\theta} y_{t+j} - w_{t+j} n_{t+j}(i) - r_{t+j}^k k_{t+j}(i) \right),
\]

subject to Dixit-Stiglitz demand constraints

\[
E_t \beta^j \left[ f(A_{t+j} n_{t+j}(i), k_{t+j}(i)) - \left( \frac{P_t(i)}{P_{t+j}} \right)^{-\theta} y_{t+j} \right] = 0, \text{ with } j = 0, 1, 2, \ldots
\]

The first order conditions for the choice variables \( P_t(i), n_t(i), \) and \( k_t(i) \) are

\[
E_t \sum_{j=0}^{\infty} \beta^j \chi^j \left[ (1 - \theta) \left( \frac{P_t(i)}{P_{t+j}} \right)^{-\theta} y_{t+j} + \theta \psi_{t+j} \left( \frac{P_t(i)}{P_{t+j}} \right)^{-\theta-1} \frac{y_{t+j}}{P_{t+j}} \right] = 0, \quad (P_t(i))
\]

\[
-w_t + \psi_t f_{n_t(i)} = 0, \quad (n_t(i))
\]

\[
-r_t^k + \psi_t f_{k_t(i)} = 0, \quad (k_t(i))
\]
where $\psi_t$ is the Lagrange multiplier of the Dixit-Stiglitz demand constraint in period $t$. The production function represents a Cobb-Douglas technology with a labor-augmenting technology shock

$$f(A_t n_t(i), k_t(i)) = (k_t(i))^{\eta} (e^{A_t n_t(i)})^{1-\eta},$$

where $0.0 < \eta < 1.0$ and $A_t$ follows an AR(1) process. It can be observed that the Lagrange multiplier, $\psi_t$, represents the real marginal cost as indicated by the first order condition of labor$^7$

$$\psi_t = \frac{w_t}{f_{n_t(i)}}.$$

Loglinearizing the first order condition for pricing, it is obtained

$$\log P_t(i) = (1 - \beta \chi) E_t \sum_{j=0}^{\infty} \beta^j \chi^j \left( \log P_{t+j} + \widehat{\psi}_{t+j} \right), \quad (13)$$

where $\widehat{\psi}_{t+j}$ denotes the log deviations of the real marginal cost from steady state. Meanwhile, the Dixit-Stiglitz aggregate price level can be computed as a weighted average between optimal prices and aggregate lagged prices, $P_t(i) = \left[ (1 - \chi) P_t(i)^{1-\theta} + \chi P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$, that can be loglinearized to yield

$$\log P_t = \chi \log P_{t-1} + (1 - \chi) \log P_t(i),$$

and combined with the definition of inflation, $\pi_t = \log P_t - \log P_{t-1}$, gives

$$\pi_t = \frac{1-\chi}{\chi} (\log P_t(i) - \log P_t), \quad (14)$$

As in Walsh (2003, chapter 5), (13) and (14) can be used together to derive the New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta \chi)(1-\chi)}{\chi} \widehat{\psi}_t. \quad (15)$$

that determines the forward-looking evolution of the rate of inflation depending on fluctuations of the real marginal cost.

### 3 Dynamic equations

The New Keynesian model with banking features described above contains three blocks: an Aggregate Demand (AD) sector, an Aggregate Supply (AS) sector and a banking sector. The AD block

$^7$The real marginal cost is the derivative of the total cost with respect to output, which can be decomposed as

$$\frac{\partial TC_t(i)}{\partial y_t(i)} = \frac{\partial TC_t(i)}{\partial n_t(i)} \frac{\partial n_t(i)}{\partial y_t(i)} = w_t \frac{1}{f_{n_t(i)}}.$$

10
depicts IS-style dynamic fluctuations of both spending on both consumption and capital goods as the endogenous determinants of expenditure-driven output. There are definitions of the marginal finance cost of consumption, $\phi_t$, and the marginal collateral service of either bonds or capital, $LSY^b_t$ and $LSY^k_t$, which affect spending decisions. The marginal finance cost affects negatively to current consumption and may also reduce investment spending as it represents the marginal cost of increasing deposits that are required for the additional purchases. As for $LSY^b_t$ and $LSY^k_t$, the marginal collateral services increase the final return on either bonds and capital. If the liquidity service yield on bonds, $LSY^b_t$, increases the opportunity cost of spending on consumption and investment goods would rise because the bond final return would be higher. Thus, $LSY^b_t$ has a negative impact on both consumption and investment. By contrast, the liquidity service yield on capital, $LSY^k_t$, increases the final return on the stock of capital and, therefore, enters the capital accumulation equation with a positive sign.

The AS sector introduces inflation dynamics driven by the standard forward-looking New Keynesian Phillips curve. Inflation evolves depending on current and expected future fluctuations of the real marginal cost. The only new element is the presence of the marginal finance cost, $\phi_t$, in the labor supply curve. A rise in $\phi_t$ would cut the amount of labor supplied by the household as a consequence of a higher cost on the funding required for purchases of consumption goods.

Finally, the monetary block incorporates some of the banking elements used in Bernanke et al. (1999) by means of a loan production function that relates the amount of loans available in the economy depending upon the stock of bonds and capital that serve as collateral. In addition, a deposit-in-advance constraint connects fluctuations of loans to those of consumption and investment. And last but not least, there are four nominal interest rates distinguishable in the model: the interbank nominal interest rate as the monetary policy instrument, the bond nominal interest rate, the interest rate on a fictitious bond that does not provide collateral services and the loan interest rate. The complete model is displayed next as the set of dynamic equations that determine short-run fluctuations:

- The AD sector consists of eight equations that provide short-run fluctuations of demand-determined output from dynamic equations that determine changes in consumption and investment. The dynamic equation that governs fluctuations of consumption was derived in Section 2 as follows

$$\ddot{c}_t = E_t \ddot{c}_{t+1} - \frac{1}{\sigma} (\phi_t - E_t \phi_{t+1}) - \frac{1}{\sigma} \left( \left( r^b_t - r^b \right) + \left( LSY^b_t - LSY \right) \right),$$

where $\phi_t$ represents the marginal finance cost of consumption displayed in equation (4). A semi-
loglinear approximation to (4) yields

$$\phi_t = \phi \left( \tilde{w}_t^m + \tilde{m}_t - \frac{\phi}{c+\tau x} \tilde{c}_t - \frac{\phi}{c+\tau x} \tilde{x}_t \right),$$  \hspace{1cm} (17)

where $\phi$, $c$ and $x$ denote the steady-state (constant) levels of the corresponding variables. Changes in the liquidity service yield on bonds $LS_{b,t}$ are given by its semi-loglinearized definition obtained from equation (6)

$$LS_{b,t} = LS_{b,t}^b \left( \tilde{w}_t^m + \tilde{m}_t - \frac{\nu k}{b+v k} \tilde{k}_{t+1} - \frac{\nu k}{b+v k} A3_t \right),$$  \hspace{1cm} (18)

where, bonds are held constant and again, $LS_{b,t}$, $b$ and $k$ denote the steady-state (constant) levels of the corresponding variables. The dynamic equation that drives capital accumulation was derived in Section 2 as follows

$$\tilde{k}_{t+1} = \frac{1}{1+\beta(1-\delta)} \tilde{k}_t + \frac{\beta(1-\delta)}{1+\beta(1-\delta)} E_t \tilde{k}_{t+2} - \frac{1+\tau \phi - LS_{b,t}^b}{\epsilon(1+\beta(1-\delta)(1+\tau \phi))} \left[ \left( r_{t}^b - r_{k}^b \right) + \left( LS_{t}^b - LS_{b,t}^b \right) \right]$$

$$+ \frac{1}{\epsilon(1+\beta(1-\delta))(1+\tau \phi)} \left[ \beta E_t \left( r_{t+1}^k - r^k \right) + \left( LS_{b,t}^k - LS_{k,t}^b \right) \right] - \frac{\tau}{\epsilon(1+\beta(1-\delta))} \left( \phi_t - \phi - \beta(1-\delta) E_t \left( \phi_{t+1} - \phi \right) \right),$$  \hspace{1cm} (19)

where the rental rate of capital $r_t^k$ is determined by the first order condition on the demand for capital, $r_t^k = \psi_t f_k(t)$, which in semi-loglinear terms becomes

$$\frac{1}{\tau x} \left( r_t^k - r^k \right) = \tilde{\psi}_t + \left( \tilde{y}_t - \tilde{k}_t \right).$$  \hspace{1cm} (20)

Meanwhile, the liquidity service yield on capital $LS_{k,t}^b$, that was introduced in (9) and also appears in (19), is close to the homonymous on bonds $LS_{b,t}^b$ as implied by the relationship $LS_{t}^k = \frac{e^{A3 t \nu (c_t + x_t)}}{b_{t+1} + A3 t k_{t+1}} = e^{A3 t \nu} LS_{t}^b$. In semi-loglinear terms, it is obtained

$$LS_{t}^k = LS_{t}^b + LS_{k,t} A3_t,$$  \hspace{1cm} (21)

where $LS_{k,t}^b = \nu LS_{t}^b$ in steady state.

Fluctuations of investment are driven by changes in the stock of capital as indicated by the loglinear version of the investment definition, $x_t = I \left( \frac{k_{t+1}}{k_t} \right) k_t$, which yields

$$\tilde{x}_t = \frac{1}{\delta} \tilde{k}_{t+1} - \frac{1-\delta}{\delta} \tilde{k}_t.$$  \hspace{1cm} (22)

Finally, the overall resources constraint that imply equilibrium in the goods market, $y_t = c_t + x_t$,
can be expressed as log deviations from steady state as follows:

\[ \hat{y}_t = \frac{c}{y} \hat{c}_t + \frac{x}{y} \hat{x}_t. \]  

(23)

- The AS sector comprises four equations that shape the behavior of firms are price setters and producers of output. The dynamic equation that determines the evolution of inflation is the New Keynesian Phillips curve that was derived in Section 2

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta)\chi(1-\chi)}{\chi} \hat{\psi}_t, \]  

(24)

where \( \hat{\psi}_t \) represents the loglinearized definition of the real marginal cost from

\[ \hat{\psi}_t = \hat{w}_t - (\hat{y}_t - \hat{n}_t). \]  

(25)

In competitive labor markets, fluctuations of the real wage can be depicted as the loglinearized labor supply curve that results from the household’s first order conditions. The supply for industrial labor services implies that \( \Xi n^x_t = \lambda_t w_t \) where the Lagrange multiplier is \( \lambda_t = c_t^{-\sigma} (1 + \phi_t)^{-1} \) as discussed above. It yields the log-linear equation for fluctuations of the real wage

\[ \hat{w}_t = \kappa \hat{n}_t + \sigma \hat{c}_t + \phi_t. \]  

(26)

Likewise, the first order condition for the supply of banking labor services, \( \Theta m^x_t = \lambda_t w^m_t \) where \( \lambda_t = c_t^{-\sigma} (1 + \phi_t)^{-1} \) can be loglinearized to reach the dynamic equation for the banking real wage

\[ \hat{w}^m_t = \kappa \hat{m}_t + \sigma \hat{c}_t + \phi_t. \]  

(27)

As for output, taking logs in the Cobb-Douglas production function introduced above for the representative firm and aggregating across firms leads to the following equation to drive supply-side fluctuations of output

\[ \hat{y}_t = (1 - \eta) \hat{n}_t + \eta \hat{k}_t + (1 - \eta) A_1 t. \]  

(28)

- The banking sector contains seven equations that determine fluctuations of loans and the variety of nominal interest rates of the model. The deposit-in-advance constraint of the households can be loglinearized to obtain

\[ \frac{c}{c+x} \hat{c}_t + \frac{x}{c+x} \hat{x}_t = \hat{l}_t, \]  

(29)
where log fluctuations of loans are provided by the log-linearized loan production function (assuming a constant level of bonds)

\[ \hat{t}_t = \frac{\alpha \nu k}{b + \nu k} \hat{k}_{t+1} + (1 - \alpha) \hat{m}_t + \frac{\alpha \nu k}{b + \nu k} A3_t + (1 - \alpha) A2_t. \]  

(30)

The real interest rate of bonds is the corresponding nominal interest rate minus expected inflation

\[ r_t^b = R_t^b - E_t \pi_{t+1}, \]  

(31)

while the real return on the physical capital is equal to the nominal interest rate on capital minus expected inflation

\[ r_t^k = R_t^k - E_t \pi_{t+1}. \]  

(32)

The central bank sets the interbank nominal interest rate, \( R_t^{IB} \). As the monetary policy instrument, the interbank nominal interest rate reacts to changes in the rate of inflation and also to deviations of the current GDP relative to its long-run trend, as prescribed by the famous Taylor (1993)’s rule. Incorporating interest-rate smoothing, the interbank nominal interest rate evolves as follows

\[ R_t^{IB} - R_t^{IB} = \mu_3 (R_{t-1}^{IB} - R_t^{IB}) + (1 - \mu_3) \left[ \mu_1 \pi_t + \mu_2 \hat{y}_t \right] + \varepsilon_t, \]  

(33)

where \( \varepsilon_t \) is a white-noise monetary policy shock. As in Goodfriend and McCallum (2007), we introduce a fictitious bond that does not provide collateral services to be the benchmark bond of a conventional New Keynesian model with no banking elements. Dropping the collateral service of bonds \( \frac{\partial L_t}{\partial b_{t+1}} = 0 \) leaves the first order condition of bonds as \( -\lambda_t (1 + r_t^T)^{-1} + \beta E_t \lambda_{t+1} = 0 \) where \( r_t^T \) is the real interest rate of the bond with no collateral capacity. If we compare this result with the actual first order condition of bonds with collateral services derived in Section 2, \( -\lambda_t (1 + r_t^b)^{-1} + \beta E_t \lambda_{t+1} - \xi_t \frac{\alpha (c_t + \pi_t)}{\beta + x_{t+1} + \xi_t} = 0 \), and also use the definitions of \( \xi_t \) and \( LSY_t^b \) from above, it is easy to reach

\[ \frac{1}{1 + r_t^b} = \frac{1}{1 + r_t^T} + LSY_t^b, \]

that can be approximated by the expression

\[ \frac{1}{1 + r_t^b} = \frac{1 + LSY_t^b}{1 + r_t^T}, \]

where taking logs and using \( \log(1 + x_t) \simeq x_t \) when \( x_t \) represents a small number such as \( r_t^b, r_t^T \) or \( LSY_t^b \), we obtain

\[ -r_t^b = LSY_t^b - r_t^T. \]
Provided that the nominal interest rate on the fictitious security is analogous to (31) as \( R_t^T = r_t^T + E_t \pi_{t+1} \), the spread that measures the collateral capacity is precisely the liquidity service yield on bonds

\[
R_t^T - R_t^b = LSY_t^b. \tag{34}
\]

The uncollateralized loan rate must coincide with the rate of return of bonds that do not provide collateral services, \( R_t^T = R_t^b + LSY_t^b \). Quoting McCallum and Goodfriend (2007): "This reflects a no-arbitrage condition between the loan market and the asset market". Therefore, \( R_t^T \) also represents the interest rate on uncollateralized loans.

Next, it is assumed that households can borrow funds from the central bank at the interbank nominal interest rate \( R_t^{IB} \) that they could lend to other households. In the case of uncollateralized loans the interest rate on those loans should take into account both the borrowing cost \( R_t^{IB} \) and the cost of producing the additional loans. The latter is determined by the marginal cost of loan production

\[
\frac{w_t^m}{m_t} = \frac{w_t^m m_t^d}{m_t^d (1-\alpha) (1+\pi_t)} = \phi_t. \quad (35)
\]

Consequently, the equilibrium condition is

\[
(1 + R_t^{IB}) \left(1 + \frac{V}{1-rr} \phi_t \right) = 1 + R_t^T.
\]

Taking logs and using again the approximation \( \log(1 + x_t) \simeq x_t \) for small numbers gives

\[
R_t^T = R_t^{IB} + \frac{V}{1-rr} \phi_t. \tag{35}
\]

Combining (34) and (35) results in the following nominal interest rate on bonds

\[
R_t^b = R_t^{IB} + \frac{V}{1-rr} \phi_t - LSY_t^b,
\]

which depend on three elements: the interbank rate, \( R_t^{IB} \), the marginal finance cost, \( \phi_t \), and the liquidity service yield on bonds, \( LSY_t^b \).

Finally, the nominal interest rate on collateralized loans, \( R_t^C \), must be lower than \( R_t^T \) because borrowers provide collateral services as owners of bonds and capital. In that case, banking activity only employs the labor cost of producing the loan. Given the loan production function at hand (with constant returns to scale), the labor cost \( w_t^m m_t^d \) is a constant share \( 1 - \alpha \) of total cost of loan production. Thus, the marginal cost of loan production is cut by \( 1 - \alpha \) and the nominal interest rate on collateralized loans, \( R_t^L \), is determined by the marginal cost equal to marginal income condition

\[
(1 + R_t^{IB}) \left(1 + (1-\alpha) \frac{V}{1-rr} \phi_t \right) = 1 + R_t^L
\]
that, after taking a log approximation, results in

\[ R_t^L = R_t^{IB} + \frac{(1-\alpha)V_p}{1-\gamma} \phi_t. \]  

(36)

Summarizing, we have a set of twenty one dynamic equations, (16)-(36), may provide solution paths for the following twenty one endogenous variables: \( y_t, \delta_t, x_t, \hat{k}_{t+1}, \hat{n}_t, \hat{m}_t, \hat{w}_t, \hat{w}'_t, \phi_t, \]

\( LSY_t^b, LSY_t^k, \psi_t, \pi_t, r_t^b, r_t^k, R_t^b, R_t^{IB}, R_t^L, \) and \( R_t^L. \) In addition, there are two predetermined variables, \( \hat{k}_t \) and \( R_{t-1}^{IB}, \) and four exogenous variables, \( A_1, A_2, A_3, \) and \( \varepsilon_t. \) This is a linear rational expectations model that may be solved using a solution method such as Paul Klein (2000)'s algorithm. The solution of the model would deliver dynamic paths for endogenous variables in which each variable would react to both predetermined variables and the exogenous variables (shocks). Such dynamic paths will be used below in Section 6 to calculate the impulse response functions that illustrate the short-run dynamic behavior of the model.

4 A New Keynesian model without banking (frictionless finance)

This section briefly describes a standard New Keynesian model with competitive labor and capital markets and without the banking elements presents in the model of Sections 2 and 3. Households are consumers and supply labor to competitive factor markets. Capital accumulation requires both a one period time-to-build delay and some adjustment costs on capital changes. There are no banking elements and purchases of consumption and investment goods do not require any holdings of deposits nor loan production.\(^9\) Adopting the same utility function specification as in the model with banking elements, the optimizing program of the profit-maximizing firm would be identical whereas the optimizing program of the utility-maximizing household would be written as follows

\[
\max_{c_t, n_t, k_{t+1}, b_{t+1}} E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{c_{t+j}}{1-\sigma} - \frac{\pi^{1+\kappa} n_{t+j}}{1+\kappa} \right]
\]

subject to current and future budget constraints

\[ E_t \beta^j [w_{t+j} n_{t+j} + r_{t+j} k_{t+j} + g_{t+j} + d_{t+j} - c_{t+j}] - I \left( \frac{k_{t+1+j}}{k_{t+j}} \right) k_{t+j} - (1 + r_{t+j})^{-1} b_{t+1+j} + b_{t+j} = 0, \]

\(^9\)Alternatively,

\[ R_t^L = R_t^{IB} + EFP_t, \]

where \( EFP_t = \frac{(1-\alpha)V_p}{1-\gamma} \phi_t \) denotes the collateralized "collateralized external finance premium", recalling the name used in Goodfriend and McCallum (2007).

\(^{10}\)For comparative purposes, this frictionless finance New Keynesian model is equivalent to the cashless economy described by Woodford (2003, chapter 1).
for \( j = 0, 1, 2, ..., \infty \). There is no deposit-in-advance requirement. The set of first order conditions includes

\[
\begin{align*}
\sigma c_t^\sigma - \lambda_t &= 0, \\
-\Xi n_t^k + \lambda_t w_t &= 0, \\
-\lambda_t I' \left( \frac{k_{t+1}}{k_t} \right) + \beta E_t \lambda_{t+1} &+ \frac{r^k_{t+1}}{n_{t+1}} \left( \frac{k_{t+2}}{k_{t+1}} \right) - I \left( \frac{k_{t+2}}{k_{t+1}} \right) = 0, \\
-\lambda_t (1 + r_t)^{-1} + \beta E_t \lambda_{t+1} &= 0,
\end{align*}
\]

where \( \lambda_t \) is the Lagrange multiplier of the budget constraint in period \( t \).

The optimizing problem of the representative firm is identical to the one described for the model with banking activities at the end of Section 2 which also incorporates Calvo-type sticky prices. The linearized equations that characterized the dynamic behavior of this New Keynesian model without banking can be organized in sectors as follows:

- The AD sector contains five equations. Using the first order conditions of consumption and bonds lead to the consumption Euler equation. After loglinearization, we obtain the standard IS-type consumption equation that brings about forward-looking log fluctuations of current consumption negatively related to the real interest rate on bonds, \( r_t \), as follows

\[
\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma} (r_t - r). \tag{37}
\]

Compared to (16), the lack of banking elements drops the marginal finance cost and the liquidity service yield on bonds out of the consumption equation. For the dynamic equation that drives capital accumulation, the first order equations on \( k_{t+1} \) and \( b_{t+1} \) are combined and loglinearized to reach

\[
\tilde{k}_{t+1} = \frac{1}{1+\beta} \tilde{k}_t + \frac{\beta}{1+\beta} E_t \tilde{k}_{t+2} - \frac{1}{\sigma (1+\beta)} \left[ (r_t - r) - \beta E_t \left( r^k_{t+1} - r^k \right) \right], \tag{38}
\]

that misses some of the elements of the corresponding equation (19) from the model with banking elements. Thus, there is no finance cost on purchases of capital goods, and the total returns on either bonds or capital do not contemplate the collateral services that these assets give in the extended model. The rental rate of capital is obtained from loglinearizing the firm’s first order condition on the demand for capital, \( r^k_t = \psi_t f_{k_t(i)} \), which after aggregation leads to

\[
\frac{1}{\tau^k} \left( r^k_t - r^k \right) = \tilde{\psi}_t + \left( \tilde{g}_t - \tilde{k}_t \right). \tag{39}
\]
Since investment is defined in an identical way to the model with banking, the dynamic equation for investment fluctuations coincides with (22) from Section 3, which is shown again here as

\[ \hat{x}_t = \frac{1}{\delta} \hat{k}_{t+1} - \frac{1-\delta}{\delta} \hat{k}_t, \]

(40)

where \( \delta \) is the rate of capital depreciation per period. The overall resources constraint indicates that total income is spent on either purchases of consumption goods or purchases of investment goods, \( y_t = c_t + x_t \), which in loglinear terms gives\(^{11}\)

\[ \hat{y}_t = \frac{c}{y} \hat{c}_t + \frac{x}{y} \hat{x}_t, \]

(41)

where \( \frac{c}{y} \) and \( \frac{x}{y} \) are the steady-state weights of consumption and investment with respect to output.

The AS sector is defined by four equations, numbered (42) through (45). Since the model includes both monopolistically competitive firms and sticky prices \( \text{a} \ la \text{Calvo (1983)}, \) the inflation dynamic equation is the New Keynesian Phillips curve (24), which is rewritten here as

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta)(1-\chi)}{\chi} \hat{\psi}_t, \]

(42)

where the loglinearized definition of the real marginal cost from \( \hat{\psi}_t = \frac{w_t}{\bar{m}(i)} = \frac{w_t}{\eta P_t(i)/n_t(i)} \) is identical across all firms in the value determined by the following expression

\[ \hat{\psi}_t = \hat{w}_t - (\hat{y}_t - \hat{n}_t). \]

(43)

The loglinearized labor supply curve is obtained from taking logs in the first order condition of labor supplied by the household, \( \sum n^i_t = \lambda_t w_t \), where the Lagrange multiplier is defined in the first order condition of consumption as \( \lambda_t = c_t^{-\sigma} \). In turn, the loglinear labor supply curve is

\[ \hat{\psi}_t = \kappa \hat{n}_t + \sigma \hat{c}_t, \]

(44)

With the same production technology as in the model with banking is equation (28) that said

\[ \hat{y}_t = (1-\eta)\hat{n}_t + \eta \hat{k}_t + (1-\eta)A1_t. \]

(45)

- The nominal interest rate and monetary policy are introduced through two additional equations, (46) and (47). There is no banking elements in the model and the only nominal interest rate is the

\( ^{11} \)In the cashless economy model, the government budget constraint is \( g_t = (1 + r^*_t)^{-1} b_{t+1} - b_t, \) which can be substituted in the household budget constraint as well as the aggregate dividends from the profit of all firms, \( d_t = \int_0^1 d_t(i)di = \int_0^1 (P_t(i)y_t(i)/P_t - w_t n_t(i) - r^*_t k_t(i)) di, \) to obtain the overall resources constraint: \( y_t = c_t + x_t. \)
one associated with purchases of bonds. Provided that \( r_t \) is the real interest rate on bonds, the nominal interest rate comes out of the Fisher relationship

\[
R_t = r_t + E_t \pi_{t+1}.
\] (46)

The central bank only role of monetary policy is to set the nominal interest rate for macro stabilizing purposes. Reproducing the Taylor (1993)-type monetary policy rule used in the model with banking, the nominal interest rate is given by the following equations

\[
R_t - R = \mu_3 (R_{t-1} - R) + (1 - \mu_3) \left[ \mu_1 \pi_t + \mu_2 b_t \right] + \varepsilon_t,
\] (47)

The set now contains eleven equations, (37)-(47), that can serve to reach solution paths for the following eleven endogenous variables: \( \hat{y}_t, \hat{c}_t, \hat{x}_t, \hat{k}_{t+1}, \hat{n}_t, \hat{w}_t, \hat{\psi}_t, \pi_t, r_t, r^k_t \), and \( R_t \). There are two predetermined variables, \( \hat{k}_{t+1} \) and \( R_{t-1} \), and four exogenous variables, \( A_1, A_2, A_3 \), and \( \varepsilon_t \). The model that represents this cashless economy with no banking activities will be solved using Klein (2000)’s algorithm and the solution paths will be employed for the computation of impulse-response functions. Results and a comparison to those of the model with banking derived in Sections 2 and 3 are provided in Section 6.

5 Calibration

Now we introduce a proposed calibration of the parameters of the model, defined for quarterly observations. This baseline calibration is oriented to use the model for the economic analysis of next sections in which we will examine the implications of introducing banking activities in the New Keynesian model, from both short-run and long-run perspectives. In that regard, some of the parameters are set at values that match realistic levels or ratios of macro variables with those obtained in the steady-state solution. Other parameters are assigned at numbers that result in reasonable business cycle properties of the model. Finally, we also borrow many numbers from the related literature.

For simplicity, the model abstracts from long-run inflation and the rate of inflation is fixed at 0% in the steady-state solution of the model. In turn, nominal and real interest rates coincide in the long-run as in Goodfriend and McCallum (2007).

To start with, let the constant discount factor be determined by both the rate of intertemporal preference (\( \rho \)) and the rate of long-run economic growth (\( \gamma \)) as \( \beta = [(1 + \rho) (1 + \gamma)]^{-1} \). Giving a 2% long-run economic growth per year (\( \gamma = 0.005 \)) and a 4% annual rate of intertemporal preference.
(\rho = 0.01) leads to a value of \( \beta = 0.985 \). It also leaves the steady-state real and nominal interest rate on uncollateralized loans at \( r^T = R^T = 0.015 \) per quarter, 6% in annualized terms. The parameters that determine the elasticities of marginal utility of consumption and labor in the utility function are set at \( \sigma = \kappa = 1.0 \) to represent the logarithmic case used in Goodfriend and McCallum (2007) and many other papers. The parameter that measures the weight of industrial labor disutility in total utility takes the value \( \Xi = 0.75 \) to imply that total labor (the sum of labor at monopolistically competitive firms, \( n \), plus labor in banking activities, \( m \)) is equal to one in steady state, \( n + m = 1 \). This normalizes total labor in a way that leaves the values of both \( n \) and \( m \) in steady-state as the fraction that represent with respect to total labor. As for the parameter on the disutility of banking labor, it takes the value \( \Theta = 18.09 \) to make the real wage on banking activities be equal to the real wage earned in other industries.

The production function is parameterized with a value of the capital share at \( \eta = 0.36 \) as assumed in the real business cycle literature (Kydland and Prescott, 1982, and Cooley and Hansen, 1989). The rate of depreciation on capital is \( \delta = 0.025 \), which implies a 10% annualized capital depreciation as also typically assumed in the real business cycle literature. The elasticity in the adjustment cost function is set at \( \epsilon = 8.0 \) in order to obtain a realistic size in the responses of investment to technology and monetary shocks.

Our choice for the parameter that determines the Dixit-Stiglitz elasticity in the demand curve is \( \theta = 11.0 \). In steady state, the value assigned to \( \theta \) conveys a mark-up of prices over marginal costs equal to \( \frac{\theta}{\theta - 1} = 1.1 \), which means that firms would charge a 10% higher price over the marginal cost. As for price stickiness, the Calvo probability is fixed at \( \eta = 2/3 \), which leads to having an average frequency of posting optimal prices equal to nine months, as suggested by the empirical evidence reported in a recent paper by Nakamura and Steinsson (2009).

The macroeconomic stabilizing role of the central bank was incorporated to the model through the Taylor (1993)-type monetary policy rule. The reaction coefficients for deviations of the rate of inflation and output from their long-run values are the ones recommended by Taylor (1993), \( \mu_1 = 1.5 \) and \( \mu_2 = 0.5 \). As for the smoothing coefficient, we set \( \mu_3 = 0.5 \) to provide enough inertial behavior of nominal interest rates that replicates the persistence observed in the data. The stock of bonds in steady state represents 42% of GDP, \( b = 0.42y \), when we mirror the bonds over consumption ratio

\[ b = \frac{1}{\kappa} y \]

Since the marginal utility of consumption is \( \frac{\partial U_t}{\partial c_t} = c_t^{-\sigma} \), the elasticity of the marginal utility of consumption with respect to changes in consumption is \( \frac{\partial (\frac{\partial U_t}{\partial c_t})}{\partial c_t} \frac{\partial c_t}{\partial c_t} = -\sigma c_t^{-\sigma-1} \frac{\partial c_t}{\partial c_t} = -\sigma \). The same treatment can be applied to obtain the elasticity of the marginal disutility of labor.
assumed in Goodfriend and McCallum (2007) to match the value observed in the US economy in the third quarter of 2005. The coefficient of reserves is $rr = 0.0218$ because that is the number found as the average of required reserves over total deposits in the US.\footnote{Goodfriend and McCallum (2007) choose a smaller reserve coefficient in their model calibration.} The velocity parameter $V$ determines the fraction of nominal deposits that is required for spending, $VD_t = P_t c_t + \tau P_t x_t$. Both $V$ and the fraction $\tau$ of purchases of investment goods that need a nominal deposit in advance are calibrated at values that provide a good match for the average ratios of total deposits over quarterly consumption and total deposits over quarterly investment observed in the US, which for the period 1980-2005 are slightly higher than 2 and around 8 respectively. Thus, we set $V = 0.2$ and $\tau = 0.75$ to jointly obtain realistic steady-state levels of deposits over consumption and investment.\footnote{Concretely, the calibrated model provides $D/P_c = 2.28$ and $D/P_x = 8.3$ in steady state.}

The three remaining parameters, $\alpha$, $\nu$ and $F$, are related to the loan production function that introduces the banking technology parameters. They are calibrated with the intention of reproducing i) a realistic banking labor share to total labor, ii) a 2.5% annualized real return of bonds and iii) a differential between the net return on capital and the return on bonds close to 3%.\footnote{This annualized 3% spread is consistent with the equity premium assumed in the finance literature.} Regarding the relative size of banking labor, the goal is to have $m = 0.04$ in the steady state solution of the model. According to the Bureau of Labor Statistics (BLS), the average ratio of the number of employees in the Credit intermediation and related activities sector relative to Total Private Employment was 0.0244 in the US economy during the period 1994-2009. On the other hand, if we considered all the employees that belong to the supersector Financial activities the percentage with respect to Total Private Employment would rise to 0.0774. Setting $m = 0.04$, we take an intermediate position between the narrow definition of banking employment used by Goodfriend and McCallum (2007) and the broad definition contained in the Financial activities supersector of the BLS.\footnote{Goodfriend and McCallum (2007) use in their calibration a small number for the banking size, $m = 0.019$, because they only take into account the employment in the industry of Depository Credit Intermediation which is a subset of the Credit intermediation and related activities.} As for the long-run rate on bonds, the historical series of real return on the annualized three-month Treasury bill over the last twenty years is 2.51\%.\footnote{We used the GDP price deflator to construct the series of inflation required to obtain the real interest rate. Source: St. Louis FRED database.} After making the calculations, the numbers chosen are $\alpha = 0.61$, $\nu = 0.44$ and $F = 10.66$. Source: FRED database published by the Federal Reserve Bank of St. Louis, which is available free of cost at the website http://research.stlouisfed.org/fred2/ .

\footnote{The series used were Required reserves and Total savings deposits plus total small-denomination time deposits. Source: FRED database published by the Federal Reserve Bank of St. Louis, which is available free of cost at the website http://research.stlouisfed.org/fred2/ .}
The following Table 1 collects all the numbers assigned in our calibration of parameters:

Table 1. Baseline calibration of parameters.

<table>
<thead>
<tr>
<th>Utility function:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption elasticity ( \sigma = 1.0 )</td>
<td></td>
</tr>
<tr>
<td>Labor elasticity ( \kappa = 1.0 )</td>
<td></td>
</tr>
<tr>
<td>Nonbanking labor weight ( \Xi = 0.76 )</td>
<td></td>
</tr>
<tr>
<td>Banking labor weight ( \Theta = 18.12 )</td>
<td></td>
</tr>
<tr>
<td>Discount factor ( \beta = 0.985 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production function and adjustment costs:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share in production function ( \eta = 0.36 )</td>
</tr>
<tr>
<td>Rate of capital depreciation ( \delta = 0.025 )</td>
</tr>
<tr>
<td>Adjustment costs elasticity ( \epsilon = 8.0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Monopolistic competition and sticky prices:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo sticky prices ( \chi = 2/3 )</td>
</tr>
<tr>
<td>Dixit-Stiglitz demand elasticity ( \theta = 11.0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Monetary policy:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation coefficient in Taylor rule ( \mu_1 = 1.5 )</td>
</tr>
<tr>
<td>Output coefficient in Taylor rule ( \mu_2 = 0.5 )</td>
</tr>
<tr>
<td>Smoothing coefficient in Taylor rule ( \mu_3 = 0.5 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Financial constraints:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposit velocity ( V = 0.20 )</td>
</tr>
<tr>
<td>Deposit requirement for investment ( \tau = 0.75 )</td>
</tr>
<tr>
<td>Reserve coefficient ( rR = 0.0218 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loan production:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale parameter ( F = 10.7 )</td>
</tr>
<tr>
<td>Banking labor share ( \alpha = 0.61 )</td>
</tr>
<tr>
<td>Capital inferiority ( \nu = 0.44 )</td>
</tr>
</tbody>
</table>

6 Impulse-response functions

The short-run analysis of the effects of banking elements in a New Keynesian model is carried out next by examining impulse response functions obtained from the four sources of variability: the production technology shock, \( A_1 t \), the monetary policy (interest-rate) shock, \( \varepsilon_t \), the banking labor
shock, $A_2t$, and the shock that alters the collateral value of capital, $A_3t$. Special attention will be devoted to the effects of this shock on the collateral value because it may well represent the economic scenario of financial crisis that we have witnessed recently. For this impulse-response analysis, it is assumed that the technology shock $A_1t$ is strongly persistent by setting its coefficient of autocorrelation at 0.95, whereas both shocks hitting production of loans, $A_2t$ and $A_3t$, are characterized by having a moderate level of inertia with a coefficient of autocorrelation equal to 0.8. The interest-rate shock, $\epsilon_t$, is a white-noise perturbation since the monetary policy rule (32) already incorporates endogenous inertia. The size of the shock is a 1% innovation. The analysis includes a comparison between three models: i) the New Keynesian model with banking elements of Sections 2 and 3 (baseline model), ii) a flexible-price version of that model obtained setting the sticky-price Calvo probability at zero and iii) the canonical New Keynesian model without financial frictions described in Section 4. The responses reported in Figures 1-4 represent percentage deviations from steady state in the case of output, consumption, investment, labor, real wage, banking labor, banking real wage and real loans, while the responses of the nominal interest rates, inflation and the liquidity service yield on capital indicate annualized deviation from the steady-state rates. Finally, the marginal finance cost is reported as the deviation from the steady-state rate (without annualizing). Each figure contains two panels; the top panel provides responses of the real variables, whereas the bottom panel displays responses of the financial variables.

### 6.1 Technology shock

Figure 1 displays the responses to a positive 1% technology shock that rises labor productivity in the goods production function. An increase in productivity reduces the marginal cost of production and monopolistically competitive firms cut optimal prices when Calvo signal allows so. In turn, the economy-wide rate of inflation falls. The reaction of the central bank to the inflation drop is announcing cuts in the interbank interest rates which are transmitted to lower interest rates on both loans and bonds (see Figure 1). The fall in the interest rates stimulates demand components, consumption and investment, and also increase the production of loans to dispose of the amount of deposits required for the purchases of additional goods. There is a higher demand for banking labor to produce more loans, and the banking real wage rises accordingly. Figure 1 shows that the three models provide similar reactions of output. Hence, the implications of the deposit-in-advance requirement and loan production are quantitatively of little importance for the business

---

19 In the case of the interest-rate shock, it actually is a 1% annualized shock.
cycle implications of technology shocks. Actually, the banking elements do not reproduce the financial accelerator of Bernanke et al. (1999) because the response of output is slightly lower when the banking elements are in place. Technology improvements increase the demand for loans, banking labor demand and its real wage, which increases the marginal finance cost, $\phi$, in a way that attenuates the response of consumption and output. This resembles the "attenuation effect" discussed in Goodfriend and McCallum (2007).

Even though the presence of banking elements attenuates the response of output and consumption, investment shows a stronger response compared to the case of a frictionless finance economy (see Figure 1). These differences are caused by the higher liquidity service yield on bonds and capital after the increase of banking labor demand to produce loans. The increase in the total return of bonds brings a more attractive compensation to savings (reducing consumption) whereas the higher liquidity service yield on capital leads to additional purchases of investment goods. In the flexible-price model, inflation falls more sharply and labor increases after the technology shock as typically observed in real-business-cycle models. The effects of the technology shock on the variety of interest rates of the model is shown in the bottom panel of Figure 1. Interestingly, bonds interest rates, capital interest rates and interbank rates all report more severe falls when financial frictions are in place. The lack of banking elements ignores the increase in the marginal finance cost, which reduces the need for a downward adjustment on interest rates to restore the equilibrium between supply and demand for goods. This has relevant consequences for monetary policy. A central bank setting the interest rate in a banking economy that oversees the role of banking and financial frictions would be cutting the prime rate by less than 40 basis points of the monetary ease recommended by a rule that takes into account banking activities. Similar gaps are observed in interest rates of assets own by the public such as bonds and capital. In the flexible-price case, the drops of all interest rates are somehow deeper to reflect the more severe decline in inflation.
Figure 1. Impulse responses after an expansionary technology shock, $A_{11} = 1.0$. 

Output, $y$ | Consumption, $c$ | Investment, $x$ | Labor, $n$
---|---|---|---
Real wage, $w$ | Banking labor, $n$ | Banking real wage, $wn$ | Real loans, $l$

FINANCIAL VARIABLES

Inflation, $\pi$ | Bond interest rate, $R_b$ | Coll. Loan interest rate, $RL$ | Uncoll. Loan interest rate, $RT$

Capital interest rate, $R_k$ | Interbank interest rate, $RIB$ | Marg. finance cost, $\phi$ | LSY on capital, $LSY_k$
6.2 Monetary policy shock

An unexpected increase in the interbank nominal interest rate set by the central bank can be induced in the model by having a positive value on the monetary policy shock to the Taylor-type rule, equation (33). The impulse response functions obtained from an annualized 1% monetary policy shock \( \varepsilon_1 = 0.25 \) are shown in Figure 2. The flexible-price case shows the neutrality of monetary shocks, the only effect of the monetary tightening is the inflation drop. All real variables remain at their steady-state levels, as well as the demand for loans. On the financial variables, the marginal finance cost, the liquidity service yield and all the interest rates have no reaction as the deflation episode wipes away the monetary contraction.

The sticky-price baseline model with financial frictions features monetary non-neutrality because the initial increase in nominal interest rates results in higher real interest rate that drive consumption and investment down. The amount of output produced and also that of real loans fall to accommodate downward demand conditions. As firms cut production, labor demand falls, productivity rises and the real marginal cost moves downward. Those firms that can set the optimal price will charge a lower price in reaction to the decreasing marginal cost. Inflation drops as a result. Since the central bank has an instantaneous reaction to inflation deviation in the Taylor-type rule (33), the actual increase in the nominal interest rate is less than the 1% embedded in the initial interest-rate shock. The role of banking elements on output and inflation are also small in the case of a monetary shock. Figure 2 displays similar-size drops on output and inflation, with slightly lower drops in the models that incorporate banking elements. The decline in both desired spending and the demand for loans brings a decrease in \( \phi \) through lower real wages and labor in the banking sector. Such lower marginal finance cost helps to contain the drop in consumption and investment.\(^{20}\) Figure 2 shows how the drop in the marginal finance cost is accompanied by declines in output, consumption, investment and labor smaller in the model with banking elements compared to the cashless economy model.

The bottom panel of Figure 2 also shows the different reactions of interest rates. In the sticky-price models, all interest rates raise after the monetary contraction except for the interest rate on capital goods which falls due to the decline in the marginal product of capital. With flexible prices, the interest rates remain unaltered because the inflation drop neutralizes the interest-rate shock leading to no change in the nominal interest rate under the application of Taylor rule. The lack of

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\(^{20}\)This again brings the idea of a financial attenuator introduced in Goodfriend and McCallum (2007), based on a procyclical response of the marginal finance cost, \( \phi \).
financial frictions mitigates the increase in the nominal interest rate compared to the model with marginal finance cost.
Figure 2. Impulse responses after a contractionary monetary policy (interest-rate) shock, $\varepsilon_1 = 0.25$. 
6.3 Labor banking productivity shock

Figure 3 provides the impulse response functions obtained in reaction to a positive 1% shock to labor in the loan production function, $A2_1 = 1.0$. There are no responses reported from the model without banking because it does not include loan production. As productivity of banking labor rises, the marginal finance cost falls and both consumption and investment increase due to such lower finance cost. Both output and inflation rise in response to this demand expansion. The quantitative effects of this shock are not very significant. Thus, output increases by less than 0.1% and (annualized) inflation rises less than 20 basis points in the baseline model with sticky prices. With flexible prices, inflation moves up 30 basis points and the increase in output is 0.03%. The demand for real loans reports an increase of similar magnitude to that of output. Both banking labor demand and banking real wages fall as a consequence of higher productivity of labor banking.

The central bank responds to the inflation pressure with higher interbank rates that are transmitted to the interest rates on both loans, bonds and capital. In the baseline model, the increase of interest rates is rather low, between 10 and 15 annualized basis points. As shown in Figure 3, the interest rates on loans rise in a more moderate way than the interbank rate because the decline if the cost of loan production (the marginal finance cost falls).

If prices are fully flexible (circle-marked lines in Figure 3), the central bank must take a more aggressive action of policy tightening as inflation rises more than in the sticky-price case. The higher interbank rates observed with flexible prices are also transmitted to the asset markets of bonds and loans.
Figure 3. Impulse responses after an expansionary shock on labor banking productivity, $A_{21} = 1.0$. 

**REAL VARIABLES**

- Output, $y$
- Consumption, $c$
- Investment, $x$
- Labor, $n$

**FINANCIAL VARIABLES**

- Inflation, $\pi$
- Bond interest rate, $R_b$
- Coll. Loan interest rate, $R_L$
- Uncol. Loan interest rate, $R_T$

- Capital interest rate, $R_k$
- Interbank interest rate, $R_{IB}$
- Marg. finance cost, $\sigma$
- LSY on capital, $LSY_k$

---

 Baseline  
 Flexible prices
6.4 Financial shock

The banking model described in Section 2 includes an exogenous process $A_3_t$ that tunes up or down the collateral value of the stock of capital for loan production. We can refer to $A_3_t$ as a financial shock in the sense of a financial source of business cycle fluctuations. A positive $A_3_t$ would make $e^{A_3_t}$ higher than one for the calculation of the amount of loans produced in the economy. The collateral value of capital would increase at any given stock. If things turned around to be negative, we could think of a contractionary financial shock that reduces the collateral value of capital, increases the cost of loan production and thus raises the marginal finance cost. Figure 4 shows the results obtained after a 1% negative financial shock, $A_{31} = -1.0$. The transmission channel from the financial shock to the real sector of the economy is the same as in the labor banking shock although now all the variables move on the opposite direction. Hence, the marginal finance cost, $\phi_t$, rises due to the decline in the collateral value of capital. The increase in the finance cost reduces the demand for both consumption goods and investment goods as indicated by the dynamic equations (16) and (19) of the model. Output falls by 0.13% (approximately 1/8 of the size of the shock), real loans drop at a similar extent, whereas nonbanking labor is cut in 0.2% units (1/5 of the shock). Banking labor and real wages move upward as a result of the substitution between loan production inputs. In the flexible-price case, the real effects of the financial shock are less important because output, labor and real loans report drops of about one half of those observed in the baseline sticky-price model.

Regarding financial variables, inflation and interest rates fall as a result of the demand contraction. The interbank rate reports a sharper decline than those of the interest rate on loans and bonds. The explanation for such spread is found in the increase of loan production (higher marginal finance cost) which does not allow a full transmission from interbank rates to asset market rates. The flexible-price model provides more substantial effects of the financial shock on inflation and interest rates than in the case with sticky prices. Intuitively, the adjustment of the excess supply occurs mostly through lower prices and interest rates and the real implications are less significant.
Figure 4. Impulse responses after a contractionary financial shock, $A3_1 = -1.0$. 
The financial shock can illustrate the effects of the credit crunch and the episode of economic recession that hit the United States and other industrialized economies in 2007-2009. The origin of the crisis was the housing bubble that was followed by a significant home price correction and the liquidity crisis. Such scenario can be incorporated in the model as a large financial shock. According to Standard and Poor’s Case-Shiller Home Price Indices, the US 10-city Composite Index declined by 33.5% from the second quarter of 2006 to April 2009. The stock market also suffered a large correction. Hence, the Dow Jones Industrial Average fell by nearly 50% from the third quarter of 2007 to the first quarter of 2009. Adopting the correction in home prices as a measure of the decline in the value of the stock of capital, we can compute in the model the responses of output, inflation and interest rates to such a financial distress. Table 2 gives the numbers obtained after a 33.5% decline in collateral value of capital, $A_3 = -33.5$, in the baseline (sticky-price) model and in a variant of the model with flexible prices. The real effects predicted by the baseline model are coherent with what has been observed in the aftermath of the crisis: output suffers a 4.24% contraction, consumption falls by 3.9%, investment declines by 5.46% and (nonbanking) labor falls by 6.62%. Table 2 also report significant effects on the financial variables. The weakness of demand brings a drop of 6.66% in the annualized rate of inflation which resembles the temporary deflation episode that experience the US economy in 2009. The model also predicts significant cuts in all interest rates. The interbank rate reaches its minimum value 6.01% below the initial rate as a result of applying the Taylor-type rule (33) in a context of rapid price corrections. The interest rates on bonds (-5.60%) and, especially, on both collateralized loans (-4.41%), uncollateralized loans (-2.14%) and capital rental (-1.40%) provide cuts of less magnitude which opens up significant spreads between the interbank rate and the other lending rates.
Table 2. Peak effects of a 33.5% decline in collateral value of capital

<table>
<thead>
<tr>
<th></th>
<th>Baseline sticky-price model</th>
<th>Flexible prices ((\eta = 0.0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-4.24%</td>
<td>-1.58%</td>
</tr>
<tr>
<td>Consumption</td>
<td>-3.90%</td>
<td>-2.58%</td>
</tr>
<tr>
<td>Investment</td>
<td>-5.46%</td>
<td>+2.08%</td>
</tr>
<tr>
<td>Labor</td>
<td>-6.62%</td>
<td>-2.46%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-6.66%</td>
<td>-14.5%</td>
</tr>
<tr>
<td>Interbank interest rate (ann.)</td>
<td>-6.01%</td>
<td>-11.2%</td>
</tr>
<tr>
<td>Bond interest rate (ann.)</td>
<td>-5.60%</td>
<td>-10.7%</td>
</tr>
<tr>
<td>Uncol. loan interest rate (ann.)</td>
<td>-2.14%</td>
<td>-6.39%</td>
</tr>
<tr>
<td>Col. loan interest rate (ann.)</td>
<td>-4.41%</td>
<td>-9.35%</td>
</tr>
<tr>
<td>Capital interest rate (ann.)</td>
<td>-1.40%</td>
<td>+0.24%</td>
</tr>
</tbody>
</table>

With flexible prices (\(\eta = 0.0\)), the real effects of the bubble burst are quantitatively smaller. As Table 2 also shows output declines by less than half of the contraction observed with sticky prices. Moreover, investment responds with a +2% unrealistic increase.\(^{22}\) Perfect flexibility on prices permit that all firms adjust downwards their selling price in reaction to the weakness of demand. The rate of inflation falls dramatically (-14.5%, which is more than double of the fall observed with sticky prices) and the central bank cuts interbank rates very aggressively. The reduction in the borrowing cost lowers also the interest rates on loans and bonds more intensely than in the model with sticky prices.

Therefore, the combination of both financial frictions and sticky prices in a New Keynesian model is necessary to obtain realistic responses of output, inflation and interest rate to a financial shock that can be of comparable magnitude to the latest bubble burst occurred in the US. The introduction of financial constraints with flexible price overpredict deflation and underpredict real effects.

\(^{21}\)A 33.5% fall in the collateral value of the stock of capital is defined by an impulse-response exercise where the financial shock is at \(A_3 = -33.5\) in the initial period.

\(^{22}\)Households sell financial assets (bonds) to buy physical assets (capital) because the interest rates on capital goods barely changes whereas there is a substantial reduction in the rate of return of bonds.
7 Long-run analysis

The model can be solved in a detrended steady state by assuming that all the exogenous sources of variability are fixed at their expected value. Hence, the four exogenous processes are at zero, $A_1 = A_2 = A_3 = \varepsilon = 0$.\footnote{It should be noticed that when $A_{1t} = A_{2t} = A_{3t} = 0$ the exponential of the shocks are $e^{A_{1t}} = e^{A_{2t}} = e^{A_{3t}} = 1.0$ to eliminate exogenous variability from the original dynamic equations in levels.} In addition, the rate of inflation in steady state is assumed to be zero ($\pi = 0.0$) as in Goodfriend and McCallum (2007) where nominal interest rates also represent real rates. It should also be recalled that all the real variables are kept at constant levels as a result of the detrending from long-run economic growth. For the baseline model derived in Section 2, the detrended steady state can be reached by solving the following non-linear system of equations:

- The capital accumulation equation from its first order condition (10)

$$R^k + (1 + \tau \phi)(1 - \delta) = \left(1 + \tau \phi - LSY^k\right) \left(\frac{1}{1 + R^b} - LSY^b\right)^{-1},$$

(48)

where we used the steady-state properties of the adjustment costs function, $I(1) = \delta$ and $I'(1) = 1$, and the coincidence between real and nominal interest rates in steady state.

- The definition of the marginal finance cost in steady state from equation (4)

$$\phi = \frac{w^m m}{(1 - \alpha)(c + \tau \delta k)},$$

(49)

where investment is equal to $\delta k$ in the detrended steady state.

- The definition of the liquidity service yield on bonds in steady state from equation (6)

$$LSY^b = \frac{\alpha w^m m}{1 - \alpha b + \nu k}.$$  

(50)

- The fact that the liquidity service yield on bonds and capital are proportional in steady state as

$$LSY^k = \nu LSY^b$$

(51)

- The steady-state nominal interest rate on the fictitious bond that does not provide collateral services becomes

$$1 + R^T = \beta^{-1},$$

(52)

which is obtained from its implied first order condition $-\lambda_t(1 + \tau^T_t)^{-1} + \beta E_t \lambda_{t+1} = 0$, noticing that the Lagrange multiplier is constant in steady state and also recalling that the nominal and real interest rate coincide in a steady state defined with a rate of inflation at zero.
- As discussed in Section 3, the relationship between the interest rate on the bond that provides no collateral services and the interest rate on the bond that serves as collateral is

\[
\frac{1}{1+R^T} = \frac{1}{1+R^b} - LSY^b, \quad (53)
\]

or, equivalently, \( R^b = \frac{R^T - LSY^b(1+R^T)}{1+LSY^b(1+R^T)} \). This expression is found when combining the first order condition for each type of bond in a steady-state scenario with zero inflation.

- In competitive markets, \( R^T \) also defines the equivalent rate of interest applied to the uncollateralized loan offered by a bank that can borrow funds from the central bank. Therefore, the marginal cost of that uncollateralized loan is obtained as the product of the cost of borrowing \((1+R^t_B)\), multiplied by the marginal cost of production that provides the external finance premium \((1 + \frac{V}{1-\phi_t})\). It gives

\[
1 + R^T = (1 + R^t_B) \left(1 + \frac{V}{1-\phi_t}\right). \quad (54)
\]

- Analogously, the interest rate on loans that are collateralized is in equilibrium the product of the borrowing cost times the marginal cost of production exclusively attached to labor banking. In steady state, it says

\[
1 + R^L = (1 + R^t_B) \left(1 + \frac{(1-\alpha)V}{1-\phi_t}\right). \quad (55)
\]

- Recalling the firm’s first order condition on the demand for labor, the equilibrium real wage in steady state is the product of the real marginal cost by the marginal product of labor

\[
w = \psi \frac{(1-\eta)y}{n}. \quad (56)
\]

- Meanwhile, the firm’s first order condition on the demand for capital implies that the net rental rate in steady state is the product of the real marginal cost by the marginal product of capital minus the rate of capital depreciation

\[
R^k - \delta = \psi \frac{ny}{k} - \delta. \quad (57)
\]

- From the firm’s first order condition on the selling price, we can derive the real marginal cost in steady state as a constant term that reflects the mark-up between prices and marginal costs

\[
\psi = \frac{\theta - 1}{\theta}. \quad (58)
\]

- The overall resources constraint in steady state is

\[
y = c + \delta k. \quad (59)
\]
- The Cobb-Douglas production function with a constant labor, a constant stock of capital and a technology shock that always remains at its expected level is

$$y = k^n n^{1-\eta}. \quad (60)$$

- Normalizing total labor at 1.0, we can split labor in steady state between working at goods-producing firms ($n$) and working on banking activities ($m$) as follows

$$n + m = 1. \quad (61)$$

- The baseline calibration of the model fixes the steady-state amount of banking labor at 4% of total labor, which implies

$$m = 0.04. \quad (62)$$

- In addition, the amount of bonds is assumed to be at 42% of total GDP in the steady state defined at the baseline calibration of the model (as in Goodfriend and McCallum, 2007), which yields

$$b = 0.42y. \quad (63)$$

- Finally, the amount of real loans in steady state is given by the following deposit-in-advance constraint

$$c + \tau \delta k = \frac{V}{1-rr} l, \quad (64)$$

where investment is written in terms of capital as $\delta k$ and $\tau, V$ and $rr$ take the numerical values assigned in the calibration.

Hence, the steady-state solution of the model can be reached by solving a nonlinear system of seventeen equations, (48)-(64), in order to find numerical values for the sixteen variables: $k, y, c, w, R^k, n, m, b, l, \psi, \phi, LSY^b, LSY^k, RT, R^b, R^{IB}$ and $R^L$. Table 3 shows the steady-state solution of the banking model as well as the steady-state solution for the model with no banking elements that is reached by solving the system under the assumptions $m = \phi = LSY = l = 0$, that would leave the interest rates as $RT = R^b = R^{IB} = R^k - \delta = \beta^{-1} - 1$ with no reference to the interest rate on collateralized loans.
Table 3. Steady-state solution under baseline calibration

<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th>NK model without banking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock of capital</td>
<td>$k = 27.81$</td>
<td>$k = 26.69$</td>
</tr>
<tr>
<td>Output</td>
<td>$y = 3.23$</td>
<td>$y = 3.26$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$c = 2.53$</td>
<td>$c = 2.59$</td>
</tr>
<tr>
<td>Real wage</td>
<td>$w = 1.95$</td>
<td>$w = 1.88$</td>
</tr>
<tr>
<td>Labor</td>
<td>$n = 0.96$</td>
<td>$n = 1.0$</td>
</tr>
<tr>
<td>Banking labor</td>
<td>$m = 0.04$</td>
<td></td>
</tr>
<tr>
<td>Stock of real loans</td>
<td>$l = 5.64$</td>
<td></td>
</tr>
<tr>
<td>Real marginal cost</td>
<td>$\psi = 0.91$</td>
<td>$\psi = 0.91$</td>
</tr>
<tr>
<td>Marginal finance cost</td>
<td>$\phi = 0.0657$</td>
<td></td>
</tr>
<tr>
<td>Liquidity serv. yield on bonds, ann. %</td>
<td>$LSY_b^k = 3.60$</td>
<td></td>
</tr>
<tr>
<td>Liquidity serv. yield on capital, ann. %</td>
<td>$LSY_k^k = 1.58$</td>
<td></td>
</tr>
<tr>
<td>Interbank interest rate, ann. %,</td>
<td>$R_{IB}^k = 0.62$</td>
<td>$R = 6.00$</td>
</tr>
<tr>
<td>Bond interest rate, ann. %,</td>
<td>$R_b^k = 2.33$</td>
<td>$R = 6.00$</td>
</tr>
<tr>
<td>Uncollateralized loan interest rate, ann. %,</td>
<td>$R_T^k = 6.00$</td>
<td></td>
</tr>
<tr>
<td>Collateralized loan interest rate, ann. %,</td>
<td>$R_L^k = 2.72$</td>
<td></td>
</tr>
<tr>
<td>Net capital interest rate, ann. %</td>
<td>$R^k - \delta = 5.18$</td>
<td>$R^k - \delta = 6.00$</td>
</tr>
</tbody>
</table>

Interestingly, the steady state in the baseline model (with banking) brings a level of capital higher than the one reached in the model without banking elements and a level of output lower. A decline in output is the result of having part of the labor force employed in the production of loans for the financial intermediation. Why does the stock of capital increase when banking intermediation is considered? The answer to this question is found in the determination of the optimal stock of capital. Combining equations (48), (52) and (53) of the baseline model, it is obtained

$$
\frac{1 - \delta + R_k^k + \tau \phi (1 - \delta) \gamma_{k^k}}{1 + \tau \phi - LSY_k^k} = \beta^{-1},
$$

(64)

where the left-hand side represents the steady-state return on next-period capital while the right-hand side indicates the household’s rate of intertemporal preference required by the household to sacrifice one unit of current consumption for future consumption. The presence of banking elements gives rise to positive levels of the liquidity service yield on capital ($LSY_k^k$) and of the marginal finance cost ($\phi$), which are factors that determine the total return on capital (left-hand
side on 64). Their influence is of opposite sign. Thus, a higher \( LSY^k \) increases the total return on capital as the denominator of the left-hand side of (64) becomes smaller. It can also be shown that a higher \( \phi \) reduces the marginal return on capital.\(^{24}\) The economic intuition behind this result is quite straightforward. If the liquidity service return on capital rises (\( LSY^k \) higher), the overall return on capital will increase. By contrast, a higher marginal finance cost on purchases of capital goods (higher \( \phi \)) will cut down the overall return on capital.

When banking participation rises (higher \( m \)), both \( LSY \) and \( \phi \) increase in steady state (see equations 49, 50 and 51). The final effect on the left-hand side of (64) is the result of combining the positive impact of a higher \( LSY \) with the negative impact that brings a higher \( \phi \). We found that the influence of \( LSY^k \) dominates over the influence of \( \phi \), which results in a positive long-run relationship between the size of banking activities and the return on the stock of capital provided on the left-hand side of (64). Such effect would create a wedge between both sides of (64) that would make the household cut down current consumption in order to raise the stock of capital (future consumption). This wedge would be closed as the additional increase in the stock of capital lowers the marginal product of capital and, likewise, the market-clearing rental rate on capital, \( R^k \), consistent with the equilibrium condition (57). Lowering \( R^k \) would eventually restore the validity of (64). The result reported in Table 3 indicates that an economy with 4% of the labor force employed in banking intermediation holds a higher stock of capital and a lower rental rate on capital than in an economy with no financial requirement (compare across columns of Table 3).

As indicated in (60), the long-run effect of banking on output can be obtained as a combination of the (positive) effect on capital, mentioned just above, and the (negative) effect on nonbanking labor proportional to the increase in banking labor. In our baseline model, the combination of the these two opposite forces result in some economic contraction due to the financial frictions. Thus, if banking and financial requirements are eliminated the steady state level of output would increase from \( y = 3.23 \) to \( y = 3.26 \) as reported in Table 3. It roughly represents a difference of 1% of output.

The long-run welfare effects of banking can be computed from its influence on the level of steady-state consumption. Table 3 informs that steady-state consumption is at 2.53 in the banking model and it rises to 2.59 in the absence of banking elements. Consumption can be obtained in the model as the difference between output and investment. Such difference in steady state is given by \( c = y - \delta k \). As the stock of capital rises due to banking intermediation, households must make a greater effort to spend income on replacing depreciated capital which would leave less

\(^{24}\) The partial derivative of the left-hand side of (64) with respect to \( \phi \) is \( \frac{-\phi (R^k + (1-\delta) LSY^k)}{(1+\phi - LSY^k)^2} < 0. \)
income left out for consumption. The connection between consumption and welfare comes from
the utility function. Following Lucas (2000), we assume that household’s utility is the measure
of social welfare, and measure the consumption equivalent between an economy with banking
activities \((m > 0.0)\) and that without the need of banking intermediation \((m = 0.0)\). Therefore,
we can provide estimates of the welfare losses associated to banking requirements as the reduction
in consumption (in percentage with respect to output) that is observed in the steady state solution
of the model. The welfare loss is

\[
100 \left( \frac{2.594 - 2.530}{3.262} \right) = 1.96%,
\]

where we took three decimals to make a more precise calculation.

The long-run interest rates found in the steady-state solution of the models with and without
banking are also reported in Table 3. The interest rates on the model without banking (and also
the uncollateralized loan rate, \(R^T\), in the banking model) are fully determined by the calibration of
the parameters on the household’s rate of intertemporal preference and the long-run growth rate.
It yields an annualized 6% interest rate in the model without banking that would be reflecting both
the prime rate of the central bank, the interest rate on bonds and also the net return on capital
goods. In the banking model, the bond interest rate is at \(R^b = 2.33\%\) per year, close to the 2.5%
suggested in the calibration procedure. The interest rate on loans is at \(R^L = 2.72\%\) when they
are collateralized and rises to \(R^T = 6\%\) in the case of the uncollateralized loan. The net return on
capital provides a rental rate minus the rate of capital depreciation equal to \(R^k - \delta = 5.18\%\) per
year which brings a spread between the equity return and the return of the risk-free bond close
to 3% as claimed in the calibration. Finally, the interbank interest rate is low at \(R^{IB} = 0.62\%\)
to reflect the little distance between the average Fed’s prime rate and the average rate of inflation
observed in US historical data.

So far, the steady-state analysis has been implemented assuming that 4% of all labor force
is employed in banking activities \((m = 0.04)\). Now, we will replicate the analysis for economies
with either a larger or smaller banking size. Provided that \(m\) can be interpreted as the fraction
of banking activities with respect to total economic activity, we have raised the value of \(m\) from
its lower bound represented by the cashless economy \((m = 0.0)\) to the upper bound defined as
one economy in which one half of total labor force is working on loan production \((m = 0.50)\).
Alternatively, we could have recalibrated the scale parameter \(F\) in the loan production function
to obtain different values of \(m\) in steady state (higher \(F\) would correspond to lower banking labor
in steady state $m$).\textsuperscript{25} Figure 5 displays the steady-state effects of such increase of $m$ on output, capital, consumption, $\phi$, $LSY^k$, various interest rates and the welfare loss. The drift at the vertical axes represents the long-run scenario of a cashless economy with no banking activities ($m = 0.0$), represented by a diamond mark. The baseline calibration ($m = 0.04$) is shown by a star mark. At first glance, there are major quantitative implications of banking on all the variables. The stock of capital increases significantly as $m$ rises up to a value around 0.3 when it starts falling. This u-inverted pattern is explained by the (positive) influence of the liquidity service yield, $LSY^k$, and the (negative) impact of the marginal finance cost, $\phi$. Figure 5 indicates that both $LSY^k$ and $\phi$ increase with a higher $m$ that results from a less efficient banking activity. At low levels of $m$, the positive influence of higher larger $LSY^k$ dominates over the negative impact of higher $\phi$, which increases the total return on capital and drives up further capital accumulation. When banking labor takes a large percentage of total labor ($m > 0.3$) the decreasing return on $LSY^k$ lead to reversing the sign on the response with lower stocks of capital as $m$ rises. rise significantly with a larger banking size $m$ (see 49 and 50 for the structural influence of $m$ respectively on $\phi$ and $LSY$). Table 4 provides the numerical values of some of the variables of the model at increasing banking size. It can be observed how $LSY^k$ increases much slower

\textsuperscript{25}Goodfriend and McCallum (2007) examine the steady-state properties of a highly efficient banking by setting a sufficiently large value to $F$ that reduces $m$ to zero.
Meanwhile, all the interest rates fall as the banking size rises except for the interest rate on the uncollateralized loan that remains constant at $R^T = \rho + \gamma$. The interest rates on bonds, collateralized loans and interbank borrowing ($R^b$, $R^L$ and $R^{IB}$) report severe cuts (to negative values) as they are negatively affected by the tightening of finance conditions with higher values of $\phi$ and $LSY^b$. The net interest rate on rental of capital goods declines with higher $m$ at a much slower path than the other market rates because it is tied to the marginal product of capital (see equation 57).

The long-run welfare cost of increasing banking size is displayed in the bottom right corner of Figure 5 and some numbers are provided in the far right column of Table 4. The welfare loss associated with banking augments with a higher $m$ because of the fall in steady-state consumption. If banking requires only 1% of total labor force the welfare loss is a permanent 0.48% of output. These numbers rapidly grow as banking size increases. Hence, we found welfare losses of 0.96% and 1.96% of output when $m = 0.02$ and in the baseline calibration $m = 0.04$ respectively. Reducing the
banking labor from 4% to 3% would increase welfare in an output-equivalent value of 0.5%. One economy where 10% of labor is employed on banking, $m = 0.10$, would suffer from a permanent welfare loss equivalent to 5.34% of output which would reach 35.7% of output if half of the population would be working at banking intermediation in the case of a poor loan production technology. In summary, the introduction of banking in the New Keynesian model brings significant welfare implications in the long-run.

Table 4. Long-run levels with alternative banking sizes.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$F$</th>
<th>$y$</th>
<th>$c$</th>
<th>$k$</th>
<th>$LSY^k$</th>
<th>$\phi$</th>
<th>Welfare loss, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>$\infty$</td>
<td>3.262</td>
<td>2.594</td>
<td>26.69</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.01</td>
<td>18.8</td>
<td>3.254</td>
<td>2.579</td>
<td>26.98</td>
<td>0.398</td>
<td>0.016</td>
<td>0.48</td>
</tr>
<tr>
<td>0.02</td>
<td>14.2</td>
<td>3.245</td>
<td>2.563</td>
<td>27.27</td>
<td>0.794</td>
<td>0.032</td>
<td>0.96</td>
</tr>
<tr>
<td>0.03</td>
<td>12.0</td>
<td>3.236</td>
<td>2.547</td>
<td>27.54</td>
<td>1.189</td>
<td>0.049</td>
<td>1.46</td>
</tr>
<tr>
<td>0.04</td>
<td>10.7</td>
<td>3.225</td>
<td>2.530</td>
<td>27.81</td>
<td>1.584</td>
<td>0.066</td>
<td>1.96</td>
</tr>
<tr>
<td>0.06</td>
<td>8.9</td>
<td>3.203</td>
<td>2.495</td>
<td>28.33</td>
<td>2.371</td>
<td>0.101</td>
<td>3.05</td>
</tr>
<tr>
<td>0.10</td>
<td>7.1</td>
<td>3.152</td>
<td>2.420</td>
<td>29.26</td>
<td>3.951</td>
<td>0.176</td>
<td>5.34</td>
</tr>
<tr>
<td>0.15</td>
<td>5.8</td>
<td>3.074</td>
<td>2.319</td>
<td>30.21</td>
<td>5.957</td>
<td>0.281</td>
<td>8.46</td>
</tr>
<tr>
<td>0.25</td>
<td>4.3</td>
<td>2.875</td>
<td>2.091</td>
<td>31.33</td>
<td>10.24</td>
<td>0.534</td>
<td>15.4</td>
</tr>
<tr>
<td>0.50</td>
<td>2.6</td>
<td>2.152</td>
<td>1.431</td>
<td>28.81</td>
<td>25.38</td>
<td>1.628</td>
<td>35.7</td>
</tr>
</tbody>
</table>

8 Conclusions

Banking activities have been incorporated to a New Keynesian model with Calvo-style sticky prices and endogenous capital. The need for banking intermediation stems from the requirement of holding deposits in advance for all purchases of consumption and a fraction of the spending on acquiring investment goods. Following Goodfriend and McCallum (2007), households act as bankers by producing loans out of the use of banking labor and their stock of bonds and capital as collateral. For such a banking model, we have derived dynamic semi-loglinear equations that determine short-run fluctuations of consumption and capital. The introduction of banking elements adds new terms on those equations that were absent in conventional New Keynesian models. Thus, consumption negatively depends on the marginal finance cost (it makes consumption more costly) and also on the liquidity services yield on bonds (it rises the return of bonds as an opportunity cost). Investment dynamics are also influenced by the marginal finance cost (negatively) and on the liquidity services
yield on capital (positively).

The impulse-response analysis shows that the introduction of banking features in a New Keynesian model does not have significant effects for the reactions of macro variables to either technology innovations (Figure 1) or monetary shocks (Figure 2). Actually, there is some attenuating effects on the responses of output, consumption and investment that results from the procyclical behavior of the marginal finance cost. For example, a contraction of output after an interest-rate shock comes with a lower financial cost that stimulates demand to partially compensate for the interest-rate hike. We have also examined the effects of two kinds of banking shocks on loan production: one shaping labor banking productivity and the other one affecting the collateral productivity of the stock of capital. We found sizeable effects of these shocks on the real sector (Figure 4). Moreover, we did replicate an scenario of a financial crisis by producing a large drop in the collateral value of capital and the reactions found were significant falls of output, consumption, investment, labor, inflation and the interest rates (Table 2). If the exercise is repeated under flexible prices, the model reports smaller real effects and an excessive drop of inflation and the interest rates.

The model was solved in steady state for the long-run analysis of banking. The results show that the presence of banking intermediation initially increases the stock of capital because of the collateral services that provide capital, but reduces both output and consumption. Subsequently, there is a permanent welfare cost of banking activities. Our results indicate that in economies where banking takes 4% of total labor force the welfare cost is estimated at 1.96% of output.

References


